# PHYSICS THROUGH EXPERIMENT 1

# EMF

## CONSTANT AND VARYING

Babu Lal Saraf Birth Centenary E-Edition

CONTRIBUTORS

B. Saraf (Coordinator)
K. B. Garg
Y. S. Shishodia
S. Lokanathan
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**FOREWORD**

Some three years ago, the University Grants Commission entrusted to me a project for improvement of undergraduate physics teaching in the colleges under their University Leadership Programme. There were almost no restrictive conditions, a thing that speaks for the boldness of the initiative of the Commission under the chairmanship of Prof. D. S. Kothari. The subsequent encouragement, co-operation and understanding shown by them, in particular by Dr. D. Shankarnarayan, has gone a long way to help us in our work.

While physics education in our country suffers from many ills, perhaps the most serious is the inadequacy and indeed the obsolescence of the laboratory for the undergraduate. There are two ways to remedy this situation. One is to equip the laboratories with a lot of new and sophisticated instruments and gadgets, expecting assimilation to develop by itself. The other is to make a definite plan of growth in experimental methods and to design simple and inexpensive experiments attuned to this purpose. We have opted for the latter alternative.

Accordingly, a major part of our initial efforts was devoted to the development of a class of experiments that will right from the beginning, make the student feel that what he does in the laboratory is extremely pertinent to his training in physics. While the experiments designed are simple, our object has been to achieve the maximum impact in the form of instruction in basic ideas as well as in technique. At the same time, we have tried to keep the cost of the equipment as small as possible so that it is within the reach of most of our teaching institutions. We wanted to make the student feel that science is quantitative measurement of physical quantities; it was therefore essential that the measurements to be made reliable enough so that faith in the validity of physical laws could be built in the mind of the learner without reservation. I have tried, from the beginning, to keep these objectives in mind while planning and designing our equipment. Only the students and the teachers can testify from experience whether my effort has been worthwhile.

Let me, in brief, describe the work we have already done on experiments. Most of the equipment described in this book has been distributed to the colleges affiliated to the University of Rajasthan and to a few institutions in other parts of the country. A set of equipment for experiments on statistical physics has also been distributed; another set on mechanics has been prepared and is to be distributed shortly. We are now working on experiments on waves; as a part of this we have made some laser units with imported tubes and home made power-supplies. Thermodynamics will be our next field of interest. At this stage, we felt that we ought to bring out a series of publications based on these experiments addressed to the student, not merely to help him do them, but also to get a picture of our overall aim. A book on experiments is, in itself, not new. Perhaps what, we hope, is novel is the attempt to tell, as far as possible, a coherent and complete story and make the student feel that the process of learning through experiment can be fun.

The burden of organizing the book fell on my colleague S. Lokanathan. He has put up an effort, to communicate in words, the scope of the experiments and what the student can observe. K. B. Garg worked with him both to initiate a write up after working with the equipment and also to assist Lokanathan throughout this effort. He (K. B. Garg and D. T. Chandwani are responsible for determining the lay out of most of these experimental boards and their multiplication for supply to the colleges. B. C. Mazumdar shared with them the responsibility for preparation of diagrams, some of them after actual measurements. I wonder if it would ever have been possible to have some of the experiments and the equipment as a physical reality without R. C. Tailor and Y. S. Shishodia, who worked on their job relentlessly and their unending endurance in taking an item of work to near perfection deserves unqualified admiration.

No words of mine can express my gratitude to Prof. D. P. Khandelwal of HBTI, Kanpur. At every stage of development-from idea to physical realization of the experiments-his suggestions have been of immense help to us. I have already mentioned the contribution of my colleagues in bringing out this volume. I would like to make a special mention of K. B. Garg, who has followed this book, quite literally, from a nebulous script to the pages of a book.

We are conscious of the errors that might have crept in, notwithstanding our best efforts and we apologise for these.

B. Saraf

## PREFACE TO BABU LAL SARAF BIRTH CENTENARY E-Edition

It was in January 1975, more than fifty years ago, the first edition of he book was published. The second edition was printed in 1978. The experiments as well as the equipment has become regular laboratory items of several institutions including IITs spread all over India. The book had received favorable reviews, including that in American Journal of Physics Vol 44, pp. 805, August 1976. The equipment has also been displayed at 1978 Oxford Conference on "Role of Laboratory in Physics Teaching", and also at International Center for Theoretical Physics, Trieste, Italy. It has also been installed at Maiduguri University, Nigeria and at an University in Zambia.

Indian Association of Physics Teachers (IAPT) was founded in 1984 and Prof. Saraf was its FIRST President. The participants at the IAPT Convention held in Jaipur in 2023, decided that a fitting tribute to Prof Saraf, on the occasion of his Birth Centenary in 2023, will be to bring out an e-Edition of the book, which should be made available without any charges to the reader.

I, as his (Prof. Saraf) former post-graduate and later a research student, a colleague and co-author of the book, took the responsibility of composing this book in Latex taking advantage of the numerous features available in it including the circuitikz, graphics, ams, etc... packages.

Shashank Capoor, a young entrepreneur and IT wizard has taken keen interest in processing the images, circuit diagrams as well the editing of the manuscript. I wonder if this e-edition would have been possible without his involvement at all stages, including the final release.

In spite of all the care that has been taken, some errors might have crept in, I take full responsibility for them, and would be grateful for these and any other suggestions beig brought to my attention so that hey may be corrected at the earliest.

Y. S. Shishodia
Email: yad.shi@gmail.com

## PREFACE TO THE SECOND EDITION

During the three years since the first edition was published the Network Boards developed by us have become regular laboratory items in several colleges and institutions spread allover India. The reports have been, by and large, quite encouraging. That the basic designs and experiments suggested are both satisfactory is borne by the fact that this second edition contains very few material changes from the first one. Appendix III-B on voltage integration and article IV-B-3 on "Representation of Complex quantities" have been rewritten, as also the write up on some experiments. But the experiments as such remain the same.

Thanks are due to N.L. Sharma and R.S. Chabra for checking up all the experiments and giving new data in several cases and to D.P. Khandelwal for final editing of the book.

B. Saraf

## PREFACE TO THE FIRST EDITION

This book is the first of the series that we plan to bring out on laboratory work and deals with experiments in electricity carried out with the networks designed by us. It is, in fact, one single and complete family of experiments that examines closely the behaviour of circuit elements with constant, varying and alternating emf. We have tried to achieve a smooth and logical transition from constant emf to alternating emf studies. In this sense, it would perhaps be most profitable to carry out these experiments in the sequence followed in this book. However, it may sometimes be difficult to adopt this pattern for one reason or another. In that case we would suggest that the teacher, to begin with, may spend a few sessions acquainting the students with the theme of these experiments preferably accompanied by demonstrations of a few of them. The students may then take up the work from different points at the same time or at different times if the experiments have to be phased out over the duration of a course.

Boards of networks described in this book are now with almost all the colleges in Rajasthan and a number of colleges outside it. Although what follows now refers to the networks designed by us it should not deter those who wish to make their own networks from using this book for performing the suggested experiments. For this reason, we have given, as far as possible, complete details of the circuits even if some of it is, quite possibly, beyond the capacity of the students to understand.

A few points about the equipment designed by us deserve special mention. These are: The choice of the parameters, the components (both as regards their quality and values and the lay out of the boards has been made so as to achieve maximum efficiency in performing the experiments. For all electrical measurements, with d-c as well as with a-c, we have used only two types of meters. All the current measurements are made with a 1mA movement meter having a resistance of about 80 ohms. After conversion, the meter has a fairly low impedance and can go in a circuit without disturbing the current distribution in the network. For all voltage measurements we have used a $50\mu A$ or a $25-0-25\mu A$ large size meter having a resistance of about $1.8k\Omega$ which, on conversion, serves as a reasonably high impedance voltmeter. For a-c measurements, the same d-c meters have been used voltmeter. For a-c measurements, the same d-c meters have been used but with the necessary bridge rectifier arrangement. All these factors have helped us to achieve proportionately low impedance and at the same time, a low power consumption in our circuits. The equipment is thus made reliable for quantitative studies.

In order, to provide a good and lasting insulation the components are all mounted on a study Bakelite sheet which has the added advantage of being a bad absorber of moisture. To make the equipment look attractive, the Bakelite sheet is topped with sun-mica coating. If this induces the student to go to the laboratory in anticipation of having some fun in learning, such small indulgence in luxury in design would be move than amply rewarded.

In this book, we have limited ourselves, to a description of experiments which seemed to us to drive home the basic ideas and are easily grasped by the undergraduate student at an early stage. However, with much the same equipment one could go on to a series of somewhat more advanced ideas. For example, we have only briefly touched on the concept of Q of resonant circuits. One could go on to experiments on filters or, for example, to multiple loops leading to ladder networks. In fact, there are any numbers of variations on the theme and at various levels. It is our hope that different institutions will develop different ideas and not wait for a common syllabus, that most tragic of all our constraints. It is in this spirit, to encourage the student to develop his own ideas, that we have addressed this book to him.

B. Saraf

## ACKNOWLEDGMENTS

A project of this kind which involves laboratory work, has to function with the active co-operation of several persons. While we mention just a few of them, we do apologise for the nearly impossible task of naming them all.

One of our colleagues on the academic staff of the project, B.C. Mazumdar, has been of great help in preparation of the drawings and making helpful comments on the script. S. R. Sharma, another colleague of ours, organized the work of distribution of the equipment to the colleges. Other members on the academic staff of the project who were of help are: N. K. Sharma, N. L. Sharma and R. S. Chhabra. We must mention A. K. Chaudhary and R. K. Rastogi who have been ably assisting us in our work. Others who have assisted in the work for short times are: A. R. Gautam, B. M. Warman, D. N. Sansanwal, K. R. Solanki, P. Kakra, R. S. Rao and V. G. Rao. We thank them all.

The workshop jobs were well executed by G. Thomas and M. L. Chawla under the active supervision of H. L. Sarna for which they deserve our warm appreciation. We thank D. D. Soni and H. C. Wadhwa for making the drawings and Y. P. Agarwal for rendering assistance in electronics. We also thank M/s. El-Tronics and Ramasheesh Electronics of Jaipur for doing a good job of multiplication of a major part of our equipment supplied to the colleges.

The unstinted co-operation and assistance that we have been receiving in administrative matters from the office of the Registrar,University of Rajasthan and from our own office (D. R. Arora, L. N. Sharma and O. P. Verma)has enabled us to devote more of our time on academic matters. We are grateful to all concerned for this.

# A NOTE TO THE STUDENT

What are you expected to learn out of an experiment? It is not a simple question to answer but we make an attempt.

**(a) Concept**

You should be able to acquire a better understanding of the physical concepts underlying the experiment e.g. if you are studying the charging of a capacitor, you should be able to understand that the charge on a capacitor grows exponentially with time. Besides, you should be able to appreciate the reversing of current in the circuit when the capacitor discharges. To take another example, let us suppose you wish to drop the line voltage to 10 volts before using it as a source. If you think, even after going through the first chapter, that all you need is any kind of potential divider, possibly a 1 k rheostat, then you simply have not grasped the concept of source impedance. You should have asked yourself, right at the beginning, what current you had wanted to draw.

**(b) Results**

Results in the form of numbers of curves are important but not quite in the way most of you usually take them to be. It is important to measure a physical quantity with as much care and accuracy as is possible with the given equipment. What is more important is that if your result is different from that expected, which it will be, very often, you should be able to identify the causes. If for example instead of $981 cm/sec^2$ you get $1000 cm/sec^2$ for the value of 'g' with a simple pendulum, you should be able to say how much of the error can be ascribed to the stop-watch, the measurement of length, the non-rigidity of the support etc. in other words, you should be able to say why in your experiment it would not be possible to measure 'g' to an accuracy better than 2%or so. If, in this sense, you understand the significance of your results, they are very important indeed. A complete experimentalist ought to give some thought as to how to improve on the results, to think of better designs, better measurement and better analysis.

**(c) Precept and Practice**

In a recent issue of a reputed Physics journal, the editor thought it necessary to remind scientific workers that physics was largely and experimental science. This is not to decry the role of theory but to re-emphasize the role of experiment. There is more to experimental physics than just "concepts" and "results". You may often discover, to your amazement, the gap between what you read in theory and what you actually do in practice. This is the beginning of your training to be an experimentalist!

Let us take a couple of examples from the experiments described in this book. If you look closely at the shunting of the current meters (ammeters) on these boards, you would discover that it does not look as it should according to the theory book. You will do well to inquire into the reasons (you need not do this necessarily before doing the experiment) and you will learn that this has been done for the safety of the meter.

Another example is the way these d-c meters are used for a-c measurements. All of them are initially either microammeters or milliammeters converted to read voltage and current in different ranges. For a-c measurements one can, in principle, use a single diode for rectification. In practice, we have had to use a bridge rectifier and the reasons are discussed in Appendix I-B. You should try, at some stage, to understand this modification as also to appreciate to what extant the scale of the meter for a-c would be linear etc.

**(d) Familiarity with Equipment**

An important aspect of your training is to develop, over a period of time, knowledge of various types of materials, components and equipment that may be used in a laboratory. When you use a diode you may learn the broad principles first. But then you may widen your awareness to find out what type of diode it is, what characteristics it has and its limitations. You may even want to know how much it costs. After all, if you make a gadget in your home as a hobby, the cost would be a very important consideration!
If you keep these basic aims in view, you will not be prey to a dull routine in the laboratory.

**Your lab work**

One of the most dismal features of the lab record book is the item called “precautions”. The student, all too often, writes them merely to add bulk to his report and more often than not, does not even observe these precautions. In writing these precautions ask yourself the following questions

a) Did I observe them?
b) Are they important enough to be mentioned?
c) Do I know what would happen if the precautions were disregarded?

If you cannot give a positive answer to these questions you are not really serious about the precautions. A book or a teacher will help often in pointing out precautions and you should get into the habit of using logic to analyze these precautions. Gradually, you will develop the maturity for working with new equipment. Remember, there may be a time when no book can help you and you may yourself be the teacher!

We have provided no table, no prototypes, for recording data nor have we instructed you to take a particular reading say “three times”. We believe that such instructions are not merely not necessary but can be positively harmful. They encourage the lazy student and restrict the freedom of the enterprising student.

A word about your preparation before doing an experiment. Naturally, the more you read, think and understand, the better you will appreciate the experiment. But we want to emphasize that it would neither be possible nor necessary to understand everything about the experiment before you start. What is much more important is to be aware of what you have not understood so that you can, at your convenience, come back to it.

One purpose of laboratory training is to develop a way of thinking that is somewhat different from the classroom one. If, after all the effort of doing these experiments, every time someone asks you about the charging of a capacitor you find it necessary to solve a differential equation, the purpose of laboratory training is lost. If, on the other hand, the theory teacher tells you that the source resistance is negligible and you insist on knowing how much is ‘negligible’, you have made the proper beginning!

Let us hope you will always keep this spirit.

**S.Lokanathan**
**Retired Professor of Physics**
**University of Rajasthan, Jaipur, India**

## A Tribute to Prof Babulal Saraf on Centenary Year

It is a pleasure for me to remember those years when we, colleagues and students, were inspired by the leadership of Prof. Babulal Saraf in the pioneering the U. L. P University Leadership Program), funded by the University Grants Commission, to stimulate Physics Education through Experiments.

The aftermath of the Sputnik in 1957 was to awaken scientists in the western world to a heightened responsibility, to raise the level of science teaching in Colleges and Universities. It produced excellent material, theory texts, for undergraduate instruction in Physics. In India Colleges and Universities had an even more urgent need, the rejuvenation of the laboratory as a place to learn from experiments, to acquire empirical skill and knowledge, not just as an appendage of Theory. Indeed such lethargy had settled in our college laboratories that the student was encouraged simply 'to read off numbers' (take readings !) and retire to calculate a result. Worse, he/she was even told that the result did not matter! To be fair, this had not always been so in our institutions. It was this situation that inspired Prof. Saraf to take as the first task under the U.L.P., the revival of the laboratory as an important element in undergraduate education.

The first set of experiments developed were in Electricity and Magnetism and others followed in the same spirit, that experimental technique required multi -fold understanding, design, monitoring performance and evaluation of their credibility.

Prof. Saraf was not just a leader! He generated ideas and was deeply involved in the nitty gritty of fabricating them and encouraging students and teachers in active involvement in laboratory education. As far as possible, many experiments were designed as to allow the student to pursue his interest rather than tie him down to a set praxis. This was the time when inspiring Lectures of Feynman had been published. Feynman, we are told, would advise teachers that if a teacher could not explain physics to a young student, perhaps the teacher has not understood it. Prof. Saraf and we, his colleagues, were devotees of Feynman. But Saraf had been a practitioner of that wisdom about exposition even earlier. Students were encouraged to ask questions in class. Younger staff, even students, were not afraid of challenging seniors on a point of science. Ideally, it was that spirit that was the goal in the laboratory to interact with him as a graduate student. Indeed the program enhanced the morale of us all, teachers and students, a memory I cherish.

It was felt that a write up of experiments developed should be placed before a wider audience in the form of a series of publications. I see that it was a wise decision if only for the very pleasant memories they recreate in me, a nonagenarian! Better still, this bringing together those volumes in e-book version is a fine tribute to the memory of Babulal Saraf commemorating his birth centenary.

**Prof. Y.S. Shishodia**
**Retired Professor**
**University of Rajasthan, Jaipur, India**

**Creation of Physics Through Experiment: A Memorable Collaboration**

It brings me great honour and pleasure to recall my long association with Prof. Saraf. During my 40 plus association with Prof. Saraf, I got a chance to interact with him as a graduate student, a researcher doing Ph. D under his guidance and as a colleague when I joined as a faculty in the Department of Physics, University of Rajasthan. Prof. Saraf was an inspiring leader, scientist and an educator.

Many of the physical processes occurring in the universe are far beyond what the human senses can perceive because of their complexity and scale. Prof. Saraf believed that the understanding of Physics concepts can be significantly improved by drawing connections between complex phenomena and something which is more relatable and easier to comprehend.

Prof. Saraf formed a cohesive team of physicists of diverse abilities for work on Physics Experiment. His team designed and developed a range of experiments in electrical and mechanical systems. Under his guidance, the team would identify the phenomena to be studies and accordingly design the experiment and the apparatus needed to study the concept in detail. If not readily available or very expensive, the team would develop the equipment needed for the experiments.

Prof. Saraf did not believe in using high tech instruments such as Vacuum Tube Voltmeter, Function Generators, Oscilloscopes etc for simple experiments. In the current book "Physics Through Experiment 1 EMF Constant and Varying" the reader would notice that the experiments are designed using much simpler devices such as high sensitivity analog meters for voltage and current measurements and home built ultra-low frequency sine-wave oscillator of range 0.1 Hz to 10.0 Hz , which was not commercially available. AF Power Oscillators and digital timers were also designed for experiments in electrical systems. The movement of voltmeter and current meter needles across the resistances and capacitors in RC circuit fed with ultra-low frequency sine wave oscillator and the visible phase difference between R and C and their measurement is a very instructive experience. The gradual decrease in the movement of the d-c voltmeter needle as the frequency is increased from 0.1 Hz to 10 Hz, where it is almost standstill at mid point is a good learning experience of transition from d-c to a-c response. The adiabatic charging of a capacitor discussed in the book has now become very important because of the necessity of keeping the energy dissipation low in high capacity Dynamic memories.

In the book "Physics Through Experiment 2 Mechanical Systems " experiments using specially designed air-track, coupled and forced oscillations and transmission line are discussed. Air-tracks using aluminium square pipes have been is use since early sixties. These tracks because of the electromagnetic damping could not be used for gliders loaded with magnets. This limited their use to physical interactions only. Prof. Saraf and his team designed triangular shaped air-tracks by gluing lucite sheets. These tracks opened a whole wide field of magnet - magnet interactions, generating a variety of potential shapes to simulated potential well, hills, energy transfer in elastic and inelastic collisions etc. Some of these experiments were presented by me in the **Plenary Lecture** delivered at the International Conference in Physics Education (ICPE) held in Duisburg, West Germany in August 1985.

A novel technique for study of amplitude and phase response in coupled and forced oscillations by use of Lissajous figures has been used. **The "Amplitude and Phase Response of a Driven Mechanical Oscillator" was adjudged best at the Eleventh Biennial Apparatus Competition held in New York, USA on January 31, 1979 and our team comprising of Prof. B. Saraf, Y. S. Shishodia and R. C. Tailor were awarded *FIRST PRIZE* for the innovative design of the apparatus and experiment.**

Prof. Saraf was very keen to obtain suggestions and comments from Physics fraternity. The equipment was, loaded in a mini bus and I accompanied him for demonstration of the equipment to physics department at Delhi University, IIT Delhi, Kurukshetra University, Punjab University, Vikram University Ujjain, T.I.F.R Mumbai , IIT Bombay, II Sc Bangalore, Nagpur University etc. The equipment was also shown at Oxford Conference on Physics Education and also at International Centre for Theoretical Physics ICTP (Trieste). The observations and suggestions of the physics fraternity were duly taken care of in designing the experiment and equipment. The experiments described in this book were developed about half a century ago. It is heartening to know that these are still in use in majority of institutions in Rajasthan and also in many of the IITs.

In the birth centenary year of Prof. Babulal Saraf, the bringing out of the e-book, making it available to a larger physics community free of cost is a rich and lasting tribute to Prof. Saraf for his contribution in the field of physics education.

**Prof. P. K. Ahluwalia**
**Professor of Physics(Retd)**
**Himachal Pradesh University**
**President IAPT**

### Prof. Babuu Lal Saraf : An Academic Leader Par Excellence

Late Prof. Babu Lal Saraf is a beckon of innovation on the horizon of Physics in India. An inspiring teacher, researcher, and innovator he envisaged Physics Classrooms and Physics Laboratories as a nurturing ground for the coming generations of both Physics Teachers and Students. The scale of his efforts was global and implementation rooted in ground realities in the classroom and laboratory.

I had an opportunity to listen to him in a refresher course held in Physics Department, University of Rajasthan, Jaipur. He established a UGC supported Centre for Development of Physics Education, where not only prototypes were innovated but manufactured and are being supplied even in present times to institutions reminding me of *Atmanirbhar Bharat*. This is a symbol of his farsightedness and a grand legacy for the physics community in India and abroad. He was a team person and encouraged team members not to lose sight of the big picture he was weaving to improve the quality of Physics education. At Himachal Pradesh University Shimla we were privileged to host his demonstration sessions. Carrying big steel trunks filed with apparatus was an awe-inspiring experience. His leadership legacy lies in the fact that all his team members were on the same side of the table.

Professor D. P. Khandelwal, Professor Babu Lal Saraf and Professor H. S. Hans as a team laid a solid foundation of IAPT, which is on the verge of completing 40 years of its existence.

Today the content of these books is helping IT generation to formulate and simulate these experiments on a PC/laptop/mobile, which is a happy consequence of taking this resource beyond laboratories.

### *Book Review : American Journal of Physics Vol 44, pp 805, August 1976*

**Physics Through Experiment, Vol I; EMF, Constant and Varying.** B. Saraf, S. Lokanathan, K. B. Garg, D. T. Chandwani, Y.S..Shishodia and R.C. Tailor. 144 pp. University of Rajasthan, Jaipur, India, 1975. ( Reviewed by S. Leonard Dart.)

One of the happy results of the University Leadership program in India is the development of indigenous teaching materials both software and hardware. Many American scientists who have had a hand in this development will be delighted to know that the program is going well and some of the tangible fruits are beginning to appear. This volume - the first of the series - is the result of a large effort at the University of Rajasthan to develop equipment and manuals for colleges affiliated with this university. The title of the volume indicates the emphasis of this effort , *Physics Through Experiment.* It is a well-series of laboratory experiments which lead the student in smooth and logical manner through the concepts of direct and alternating current circuits. The subtitle *EMF , Constant and Varying* describes the emphasis.

Indian universities have had a strong emphasis on laboratory work, holding practical examinations more commonly than in the U. S. However, the emphasis is generally on getting the right result quickly using unimaginative equipment. It is refreshing, indeed, to find an attempt to tell, as far as possible , a coherent and complete story and to make student feel that the process of learning through experiment can be fun. This book is far more than a set of instructions applied to some specific apparatus. It is a logical development of ideas presented with clarity and good humor. " A note to the student" sets the tone with discussion of the approach to the laboratory as the " training of an experimentalist ". A thoughtful open ended approach is encouraged . It has somewhat the flavor of the Berkeley Physics Course although the details are very different.

The general format of the chapters is a development of the concept, a description of the apparatus and then a series of experiments based on the concept. The theoretical development makes good use fo calculus and vector notation. The book follows the conventional sequence of development. The first chapter, on power sources, involves a study of source and load resistances , power delivered and load matching. Is followed by an Appendix describing in detail a regulated dc power supply. In my opinion this appendix should have been placed later in the book after feedback had been studied. There follows chapters on capacitors and RC circuits including both transient and steady state conditions. These are approached with increasing mathematical sophistication including vector representation, feedback and oscillators. Chapter VI introduces inductive circuits, oscillations, resonance and Q. The seventh chapter is on phase measurement by superposition and final chapter is on electromagnetic induction including Lenz's law and Faraday's law.

Although this book was designed as the laboratory manual for the equipment developed at the University of Rajasthan, it should find much wider use both within India and elsewhere. The equipment is carefully described and is built with components available anywhere. THE order and level of development makes it well suited to physics or engineering students at the first or second year college level.

I hope that the subsequent volumes in this series - to include statistical physics, mechanics, waves, and thermodynamics - will be of similar quality and will be completed soon. Not only Professor Saraf and the Physics Department of he University of Rajasthan to be congratulated but also the University Grants Commission for its recognition and support of this project.

*S. Leonard Dart is Professor of Physics in the Joint Science Department of the Claremont Colleges. He has been to India a number of times as a consultant, Coordinator and staff member under the Science Education Improvement Program administered by the National Science foundation*

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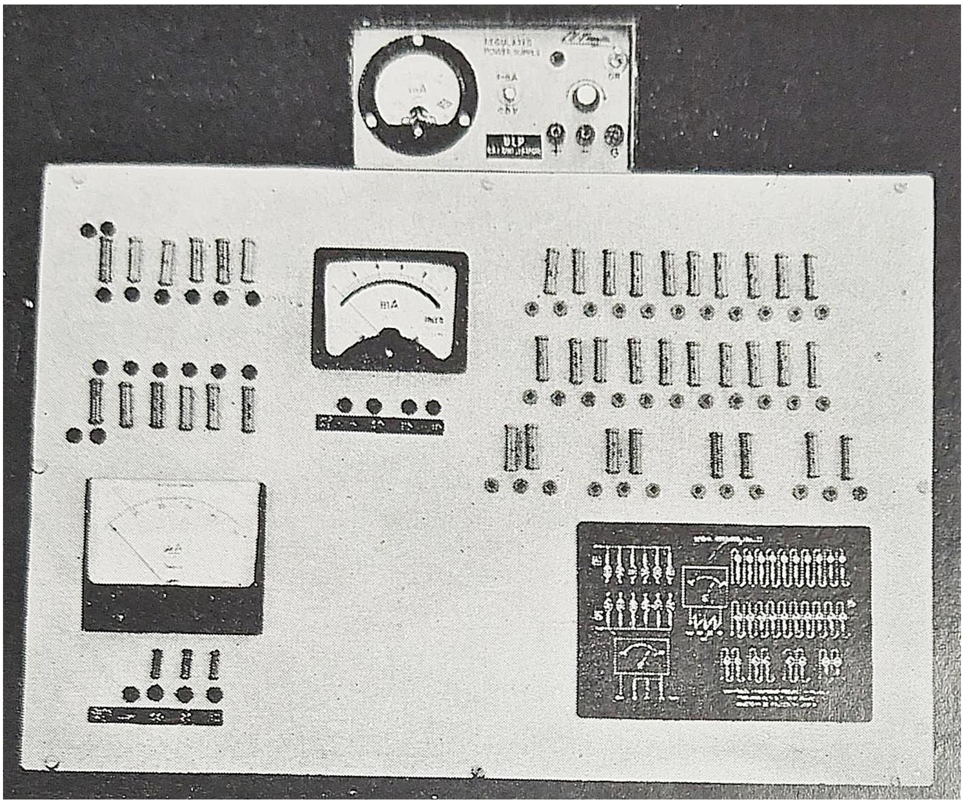

FIGURE 0.1 Network Board for Study of a Power Source

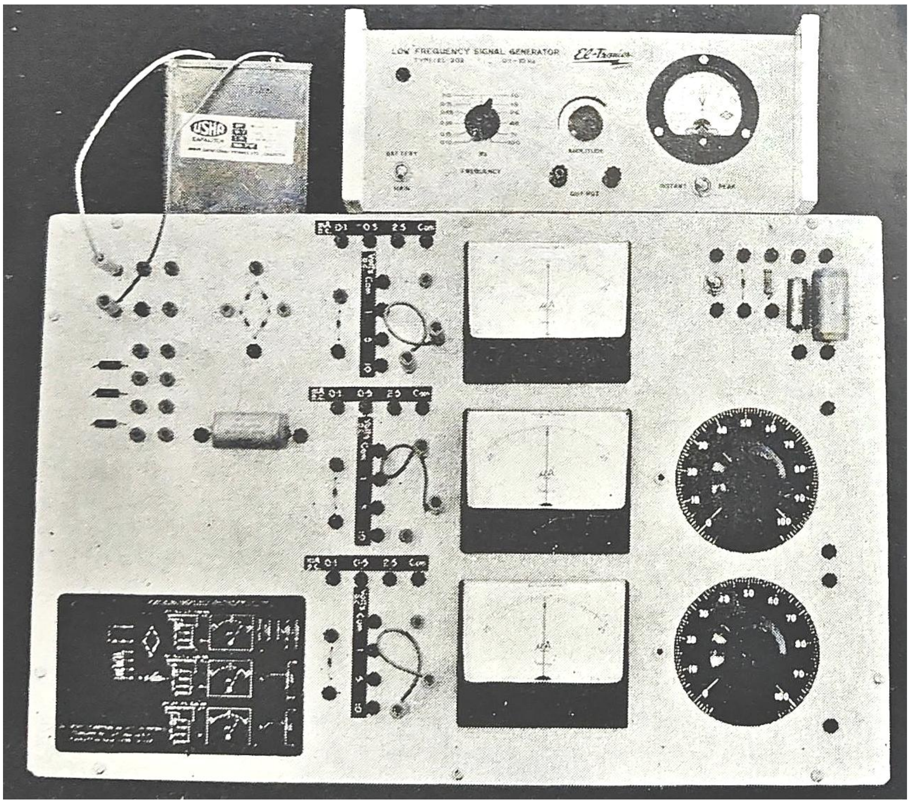


FIGURE 0.2 Network Board for Study of RC Circuit with Ultra-low Frequency a-c

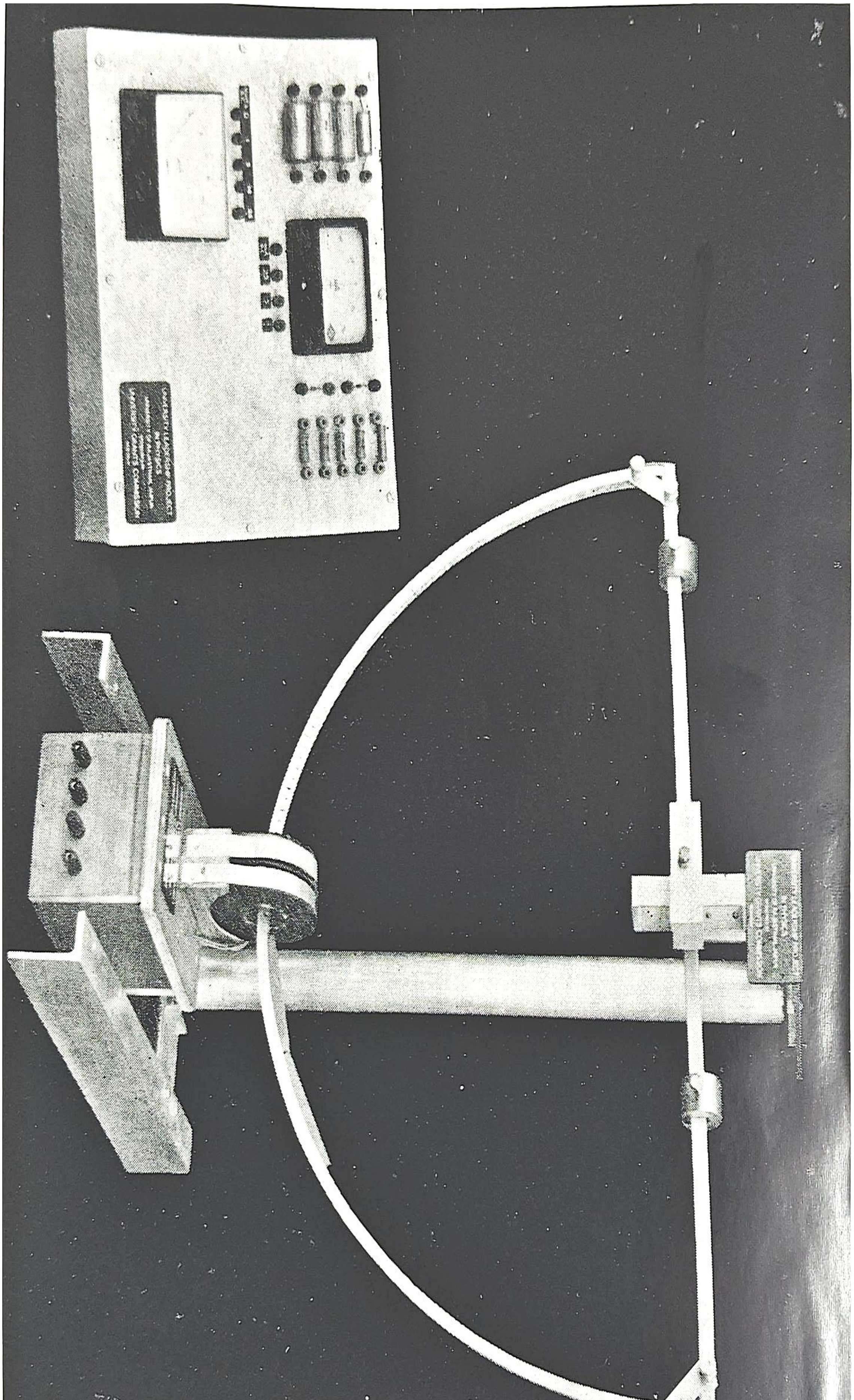

FIGURE 0.3 Apparatus for Study of Electromagnetic Induction

# CHAPTER 1

# 1. Study of Power Source

## 1.1 Introduction

A car battery can supply 12 volts. So can 8 dry cells in series. But no one would consider using the dry cells to start a car. Why not? Obviously, the dry cells cannot supply the large current required to start the car. The point is that the *resistance* of the source for the car battery ($\approx 0.1 ohm$) is considerably smaller than that for the 8 dry cells ($\approx 5$ *to* 70 ohms) in series. A power-supply which happens to be another commonly used source in the laboratory has a widely varying resistance; for a regulated power-supply it may be as small as 0.01 ohm.*

## 1.2 Output Resistance

A source of emf, therefore, must be represented not by just its voltage $V_o$ [Fig.1.1(a)], but by its *source resistance* $R_S$ as well [Fig.1.1(b)].

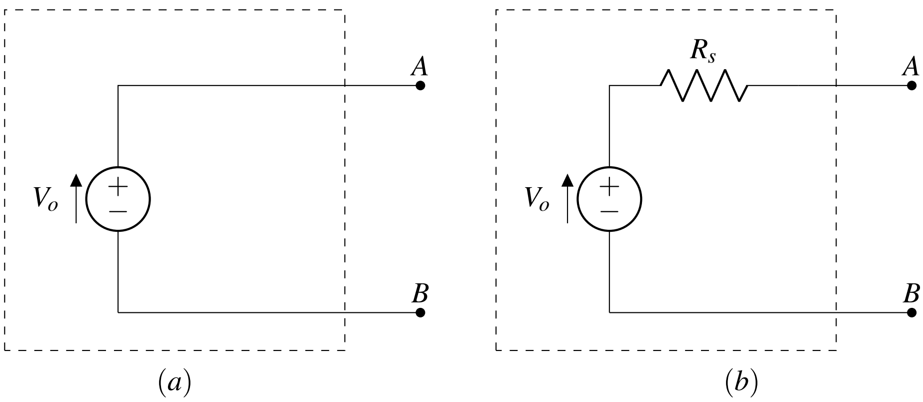


FIGURE 1.1

It is convenient to think of the source $V_S$ and its resistance $R_S$ as enclosed in an imaginary box [indicated by the dotted line in Fig. 1.1(b)] with terminals A and B, which we can put to any use we like. Electrical networks may be complicated but it is often very useful to think of parts of it as a "box" with certain parameters associated with it - in the above case the parameters being $V_S$ and $R_S$.

Suppose we are given such a box with terminals A and B and we have to determine $R_S$ and $V_S$. First let us see how to do this *in principle*. We connect a voltmeter of very high resistance (ideally infinite) so that it draws no current. It will measure $V_S$ directly. We can now connect an ammeter (ideally zero resistance) and measure the current which will be i=$V_S/R_S$.

Thus, we may *define* the source resistance as the *open circuit* voltage between A and B divided

*There are, of course, many other factors that dictate practical use of a power source. consideration of cost, convenience of use, rechargeability, available power and energy etc. are some of these. For example, a dry cell may give only a few watt-hours of energy and cannot be recharged whereas a car battery can give 500 watt-hours and, with care, can be recharged any number of times. A power-supply, on the other hand, derives its power continuously from the a-c mains and hence needs no charging and can deliver any amount of energy. We shall, however, not discuss these factors here, important as they are.

by the current when A and B are *short circuited*. In practice, we may have to exercise caution since the short circuit current may be very large and damage the instrument or the source itself.

We may now adopt a more general attitude. The terminals A and B provide a certain source of voltage $V_S$ with source resistance $R_S$. Actually, $R_S$ may include other circuit elements as well. For example, think of the arrangements in Figs.1.2(a) and 1.2(b). For these too we can represent the 'source' by a certain output voltage $V_S$ and source resistance $R_S$ as in Fig. 1.1(b).

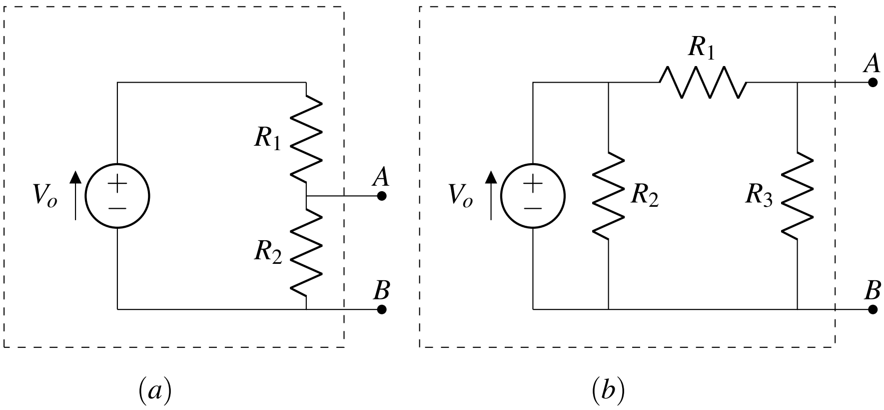


FIGURE 1.2

For the case of Fig. 1.2(a) Ohm's law gives us

$$V_S = V_0 * \frac{R_2}{R_1 + R_2}, R_S = \frac{R_1 R_2}{R_1 + R_2}$$

We can now say that across terminals AB, we have a source of output voltage $V_S$ with an effective resistance $R_S$. This effective source resistance is often called the *output resistance* of the device as seen from AB. We shall develop the above ideas with a few simple experiments.

## 1.3 The Network Board

The network-board for our experiment is shown in Plate I. It contains three group of resistors $R_1$, $R_2$, $R_3$, each group having several different resistors to choose from. It has a dc milliammeter and a d-c voltmeter.

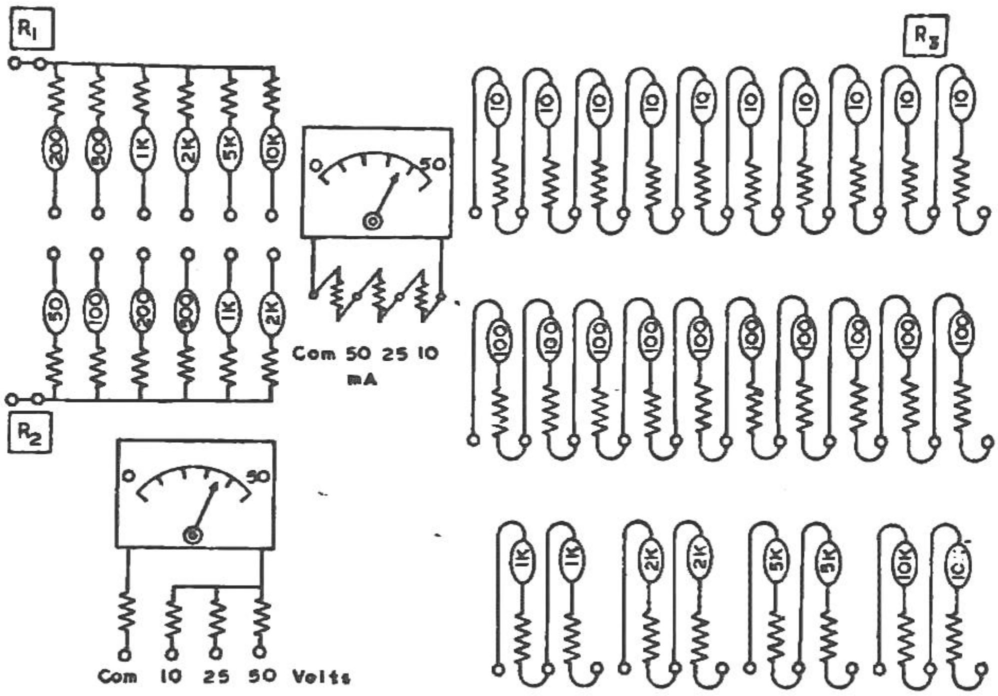


FIGURE 1.3 Network Board for Study of Power Source

Fig.1.3 shows the details of connections provided underneath the board. It will be seen that one could choose any one resistor from group $R_1$ and any one from group $R_2$ to make up a 'source' like that in Fig 1.2(a). Resistors in the third group $R_3$ are all connected in series and can be used as load. One could plug-in at any pair of points and get the desired value of the load.

The meters can be used in multiple ranges. See Appendix I-B for the shunt arrangements in case of the milliammeter and series resistor arrangements for the voltmeter. It is worth mentioning that the voltmeter has been made out of a microammeter with its microamperes scale left uncovered and the voltage reading scales marked separately below the meter. The advantage is that one an also read the current drawn by the voltmeter. In principle, for an ideal voltmeter, this should be zero; in practice it is not. For about 10 mA current in the circuit the voltmeter provided on the board draws $50\mu$A for a full scale deflection, which is about 0.5 percent.

## 1.4 Experiments

### 1.4.1 Output voltage and resistance of a source

#### Experiment I-A: To obtain the output voltage and output resistance.

Let the dotted 'box' in Fig. 1.4 with AB for its output terminals be our 'source'. As can be seen from from the figure, in fact it consists of a power-supply of voltage $V_0$ and a potential divider arrangement made of resistors $R_1$ and $R_2$. We have to measure the output voltage $V_S$ and output resistance $R_S$ of this source.

$V_S$ is measured by connecting the voltmeter directly across AB. Of course, it is implied here that the resistance of voltmeter is so large that the current flowing through this can be neglected.

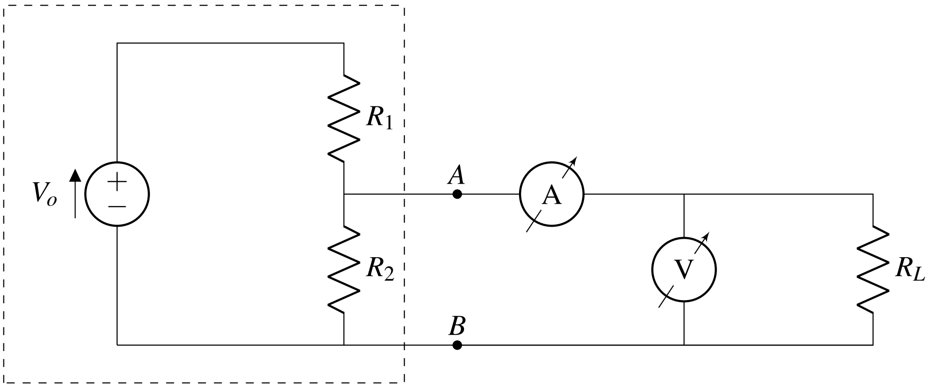


FIGURE 1.4

Now connect resistor $R_L$ called the load resistor along with a milliammeter. The current i drawn from the source is measured by the milliammeter and the new voltmeter reading $V_L$ would be lower than $V_S$. If $R_S$ be the output resistance of the source then

$$V_S - iR_S = V_L$$

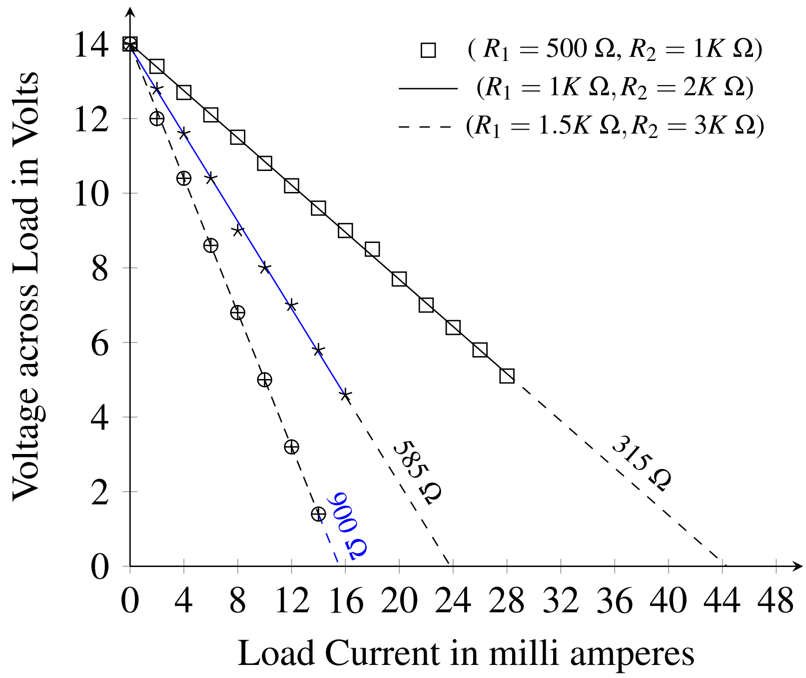


FIGURE 1.5

Thus the output resistance $R_S$ is given by

$$R_S = \frac{(Drop\ in\ output\ voltage)}{Load\ current} = \frac{V_S - V_L}{i} \tag{1.1}$$

If we take several different values of $R_L$, we shall be drawing different currents i. The voltage drop $V_S - V_L$ will also correspondingly change. You may tabulate these values, compute $R_S$ each time from Equation (1-1), and obtain the mean $R_S$. Alternatively, you may draw a graph between $V_L$ and i as shown in Fig. 1.5, see if it is a straight line, and obtain $R_S$ from its slope and $V_S$ from its intercept on the $V_L$ axis.( Since i=0 for this intercept $V_S$ would be the same as $V_L$). Can you appreciate why it is much better to deduce $R_S$ from the graph rather than directly from your observations?

Represent your results $V_S$, $R_S$ with a diagram like that in Fig. 1.1(b). This would be the 'equivalent circuit' for the actual source in Fig. 1.4.

### 1.4.2 Variation of output resistance

#### Experiment I-B: To study the variation of the output resistance $R_S$ with changes in value of $R_1$ and $R_2$, the ratio of $R_1/R_2$ remaining constant.

In the arrangement of Fig. 1.4. if the power-supply itself has resistance zero, then by Ohm's law the output voltage should be

$$V_S = V_0 \frac{R_2}{R_1 + R_2} \tag{1.2}$$

You may check the *measured* $V_S$ against the value calculated from Equation (1.2). Equation (1.2) shows that if both $R_1$ and $R_2$ are are changed by the same factor, $V_S$ does not change. The value of $R_S$ should, however, change. Let us examine this by *experiment*.

With one set of $R_1$ and $R_2$ measure $R_S$ as in Experiment I-A. Now change the resistors to $xR_1$ and $xR_2$, where x is some common factor. Measure $R_S$ again. Repeat this with different values of x and examine how $R_S$ varies with x. The variation of $R_S$ with x can be discussed as below. Figs. 1.6(a) and 1.6(b) are equivalent.

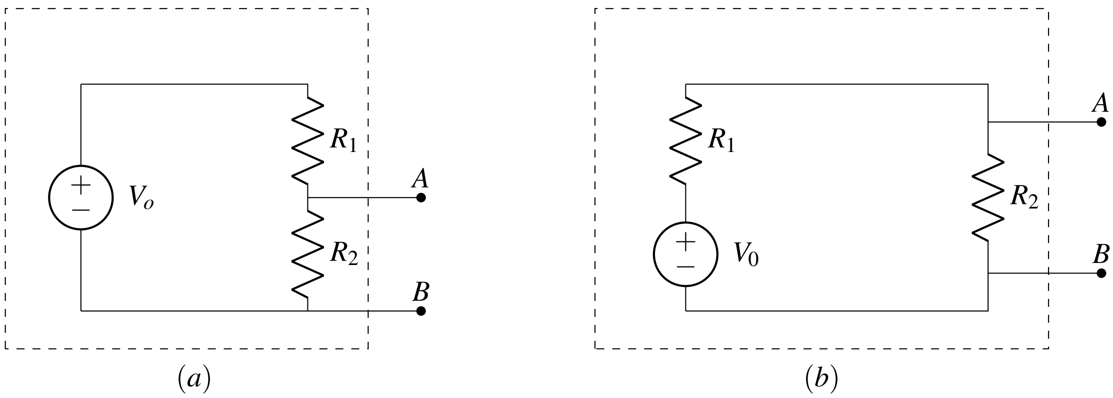


FIGURE 1.6

One can now see from the latter that if the power-supply itself has negligible resistance, then inside the 'source' $R_1$ and $R_2$ are in parallel, so that the output resistance of the source as seen at AB should be

$$R_S = \frac{R_1 R_2}{R_1 + R_2} \tag{1.3}$$

By changing both $R_1$ and $R_2$ by a factor x, the new output resistance $R'_S$ will be given by

$$R'_S = \frac{xR_1 xR_2}{xR_1 + xR_2} = \frac{xR_1 R_2}{R_1 + R_2} = xR_S \tag{1.4}$$

Thus for a given ratio of $R_1/R_2$ i.e. for a given ratio of $V_S/V_0$, the output comes out to be proportional to x.

Using Ohm's law, deduce an expression for the current i drawn by a load $R_x$ connected across AB in Fig. 1.6(a). Use this result to obtain expressions for $V_S$ and $R_S$.

### 1.4.3 Power delivered at different loads

**Experiment I-C: To study the power delivered by a source at different loads.**

A load resistor, connected across the output terminals of a 'source', draws some current from it and thus consumes the power delivered to it by the 'source'. It is interesting to study how the latter varies with the load. If, for a certain load $R_L$, the current is $i$, the voltage across the load is $V_L$, then the power P delivered by the 'source' is given by

$$P = V_L i \tag{1.5}$$

Connect different resistors $R_L$ and measure the current $i$ and voltage $V_L$ each time (Fig. 1.7). Tabulate these data and compute the power P using Equation (1-5).

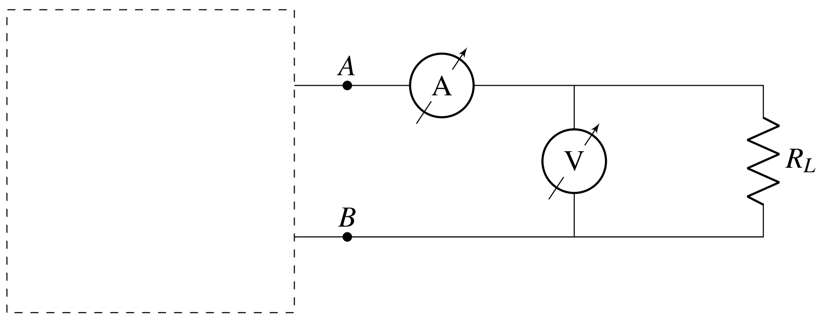


FIGURE 1.7

Also plot P against $R_L$ and draw a smooth curve through the observation points(Fig. 1.8).

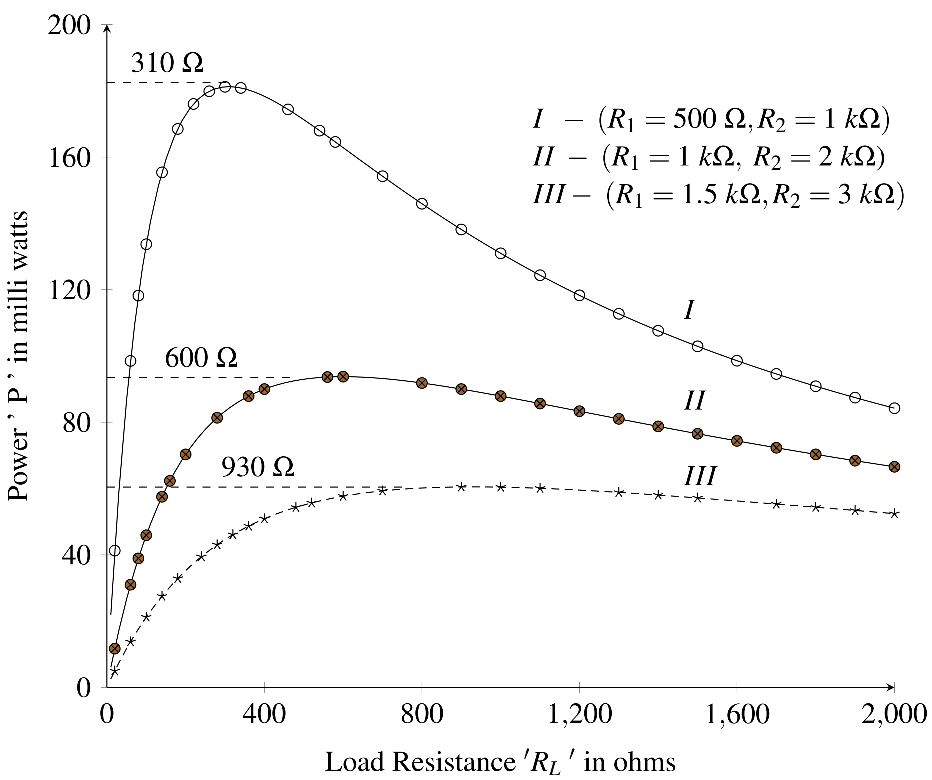


FIGURE 1.8

The curve has a broad maximum for some value of $R_L$. What is so special about this particular load? If you measure the output resistance $R_S$ of the source Experiment I-A) you will find that the power delivered is maximum when the load $R_L$ has the same value as $R_S$.

### 1.4.4 Load matching and power dissipation

#### Experiment I-D: To learn more about 'load matching' and power dissipation in a circuit.

We have already seen in Experiment I-B how for a given ratio of $R_1/R_2$, the output voltage $V_S$ does not depend on the individual values of $R_1$ and $R_2$ whereas the output resistance does. Using this knowledge try different arrangements of $R_1$ and $R2$. Measure the output resistance in each case (Experiment I-A). Also measure the variation of the power P delivered to the load $R_L$ for each value of $R_S$.

Plot the following quantities as a function of load; (a) the load current $i$ (b) the voltage $V_L$ across the load (c) the power dissipated in the load (d) the *fraction* of power dissipated in the load upon the power expended by the source. You will see that the maximum power in each case is delivered when the load $R_L$ is equal to the output resistance $R_S$. This disarmingly simple result is of great importance and you will come across it again and again in various forms. The idea is that the load resistance should 'match' the output resistance for maximum power transfer[†] You will also notice that the current $i$ is maximum when $R_L$ is zero, the voltage $V_L$ across the load is maximum when $R_L$ is infinite but the power $iV_L$ dissipate in the load is maximum when $R_L$=$R_S$ and is equal to $\frac{V_S^2}{4R_S}$. Remember, this is not the power expended by the source which is $\frac{V_S^2}{2R_S}$.

A source of output voltage $V_S$ and output resistance $R_S$ when connected across a load $R_L$ gives a current

$$i = \frac{V_S}{(R_S + R_L)}$$

and delivers power to the load given by

$$P = i^2 R_L$$

Show, using differential calculus, that P is maximum when $R_L$=$R_S$

### 1.4.5 Reflected load resistance

#### Experiment I-E:[‡] To study the reflected load resistance in a network.

Consider Fig. 1.9(a). When $R_L$ is not connected, let the current through the circuit be $i$.

[†]This statement is true for alternating current circuits also. There we talk of *output impedance* instead of output resistance and the principle assumes its general name-the principle of *impedance matching*

[‡]For doing this experiment you will need a resistance box in addition to the Network-Board shown in Fig.1.3. Also note that in all these experiments a power-supply with a negligible output resistance is used as a source.

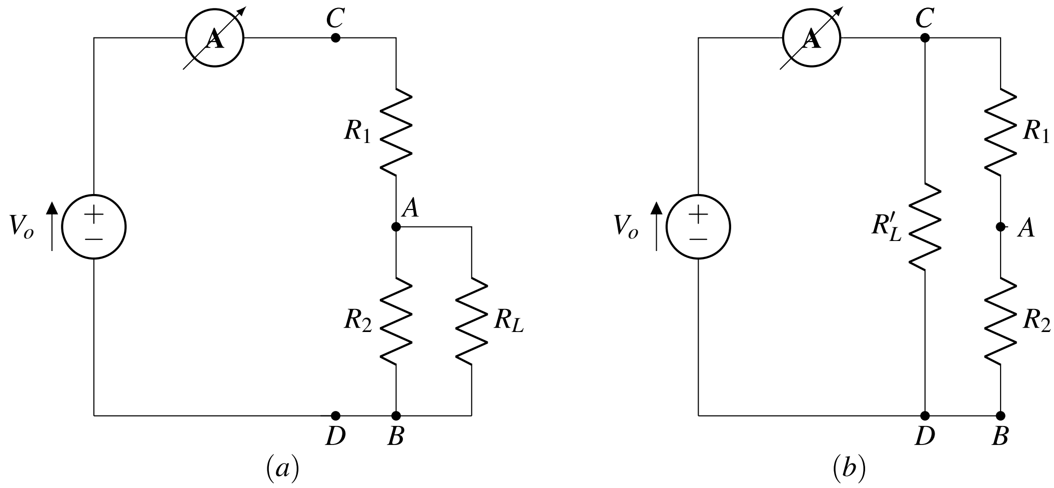


FIGURE 1.9

On connecting $R_L$, this current increases to some value $i_0$ which means that the load $R_L$ connected across AB increases the current from i to $i_0$.

We can achieve the same result if we connect a suitable load $R'_L$ across CD (which means directly across the power-supply). This load $R'_L$ seen by the source is called as the *reflected load resistance*.

For the simple circuit shown in Fig. 1.9, you can also calculate the reflected load resistance $R_L$ by applying Ohm's law but in more complex networks such calculations may not be all that simple. Nevertheless, the fact remains that for a load $R_L$ across any two points (AB) in a network, simple or complex, you can always determine the reflected load $R'_L$ as seen by the power-supply (across CD). In this sense therefore, the network acts as a 'transformer". It is usually called an *impedance transformer* when the network has components other than pure resistance also.

More generally, we can replace the actual load $R_L$ by a load $R'_L$ in another part of the circuit such that the current drawn from the source is the same. One then calls $R'_L$ as the *transfer load*( transfer resistance or transfer impedance as the case may be). An example of this is shown in Fig.1.10.

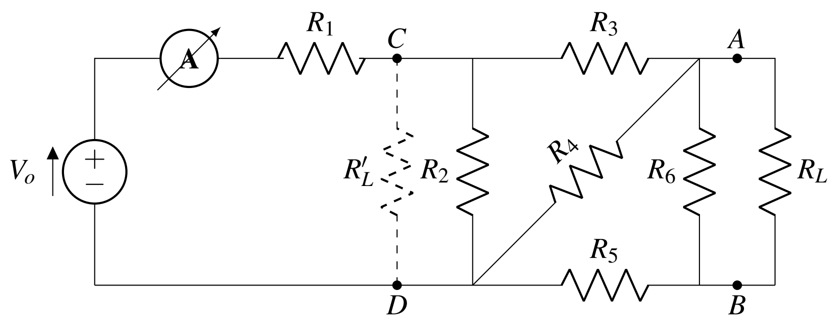


FIGURE 1.10

Deduce an expression for the current in Fig. 1.9(a) when $R_L$ is connected. Deduce a similar for the case of Fig. 1.9(b) when $R'_L$ is connected. Hence obtain an expression for the reflected load resistance. Draw conclusions for the limiting cases of $R_L \to \infty$ and $R_L \to 0$.

In the study of complicated circuits impedance transformations lead to considerable simplicity of analysis and are widely resorted to. We should, therefore, try to see this at least in a simple circuit like the one shown in Fig.1.11.

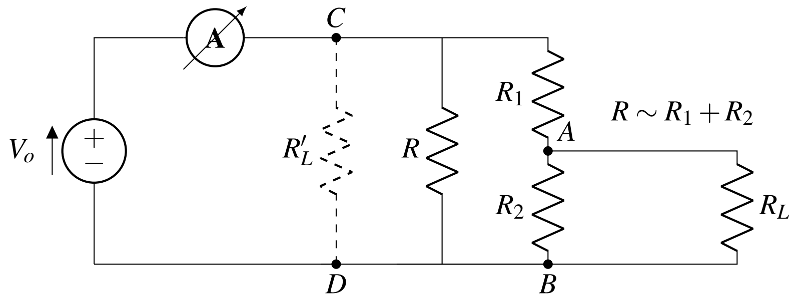


FIGURE 1.11

Use a resistance box for $R_L$ along with the network board for this experiment. First keep $R'_L$=∞ (plug off) and read the current, in the millli-ammeter when $R_L \rightarrow \infty$ and when $R_L$ has a given value. Let these readings be i and $i_0$. Now set $R_L$=∞, plug-in $R'_L$ and adjust its value such that the current has a value $i_0$. Read the value of $R_L$ at this stage.

Keeping $R_1$, $R_2$ and $R$ unchanged, take different values of load $R_L$ and for each case experimentally obtain transfer load. It may be worthwhile to plot $R'_L$ against $R_L$ and see how it varies with $R_L$.

In the circuit of Fig.1.9(a), the load $R_L$, on being connected across the output terminals AB, increases the current from i to $i_0$. Show that the transfer load (reflected resistance) $R'_L$ as seen at CD is given by

$$R'_L = \frac{V_0}{i_0 - i}$$

where $V_0$ is the output voltage at AB. (Thus, $R'_L$ can be deduced from measurements i, $i_0$ unlike the method of direct substitution suggested in Experiment 1-E).

### 1.4.6 Equivalent circuit for a source

#### Experiment I-F:[§] To make a simple equivalent circuit for a power 'source'.

In Experiment I-A you took a simple source (Fig. 1.4) of output voltage $V_S$ and output resistance $R_S$. Now you may take a far more complicated arrangement, like the one shown in Fig. 1.12(a) and measure its $V_S$ and $R_S$ as seen at the output terminals AB. Now make the simple arrangement of Fig. 1.12(b) in which voltage of the true source is adjusted to be $V_S$ and the series resistance is adjusted to be $R_S$. This then is an 'equivalent circuit' corresponding to the circuit of Fig.1.12(a). We may check this equivalence directly by experiment.

[§]This could be done immediately after Experiment 1-A as an exercise to see how any 'source' (with whatever complicated details) can be replaced by an equivalent circuit of an emf and a series resistance $R_S$. You would need some extra resistors in addition to your Network-Board for doing this experiment.

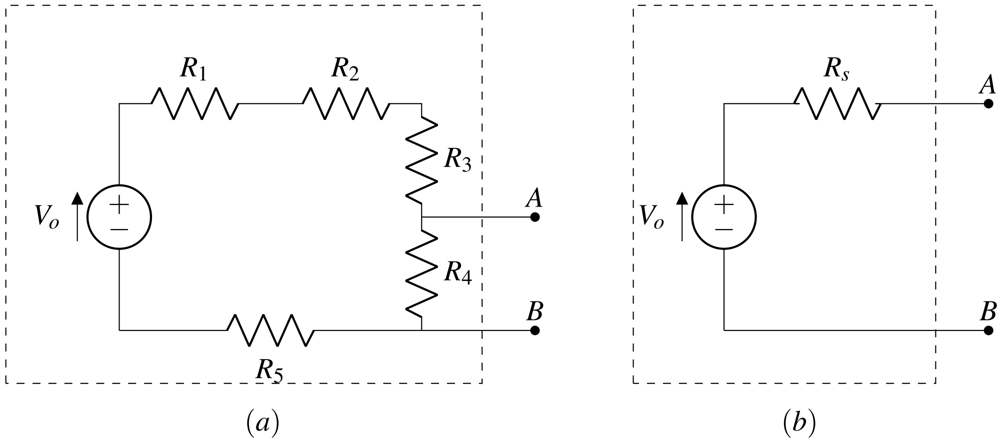


FIGURE 1.12

Make any network and choose any two points AB in that network as the 'output terminals'. Apply different loads $R_L$ at these terminals and each time measure the current i drawn and the voltage $V_L$ across AB. From these calculate $V_S$ and $R_S$ as in Experiment 1-A. Now take a power-supply and adjust its voltage to the value $V_S$. Connect a resistor of the value $R_L$ in series with it [Fig.1.12(b)]. Then apply the same loads $R_L$ across its output terminals AB and each time measure i and $V_L$. Compare these results with those obtained with the complicated network and see if the equivalence is complete.

Even when there is more than one source of emf in the network, the equivalence holds. In a-c circuits, with inductors and capacitors also present, the equivalence involves some more details, but is still a very useful concept.

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## 1.5 APPENDIX

### 1.5.1 Appendix I-A REGULATED D-C POWER-SUPPLY

An ideal regulated supply is an electronic circuit designed to provide a predetermined d-c voltage, which is independent of the load current $I_L$ drawn from it, the temperature and variations in the a-c line-voltage.

An unregulated power-supply consists essentially of a transformer, a rectifier and a filter. There are several reasons why such a power-supply is not good enough for many applications. The first is its poor regulation; the output voltage is not constant as the load varies. The second is that the d-c output voltage varies with the a-c input. In some locations the line-voltage (of nominal value 230 volts) may vary from 180 V to 250 V and yet it is necessary that the d-c voltage remain constant. The third reason is that the d-c output voltage may vary with temperature. A feedback circuit is used to overcome these shortcomings. Such a system is called a regulated power-supply [shown in Fig. I-A (1)].

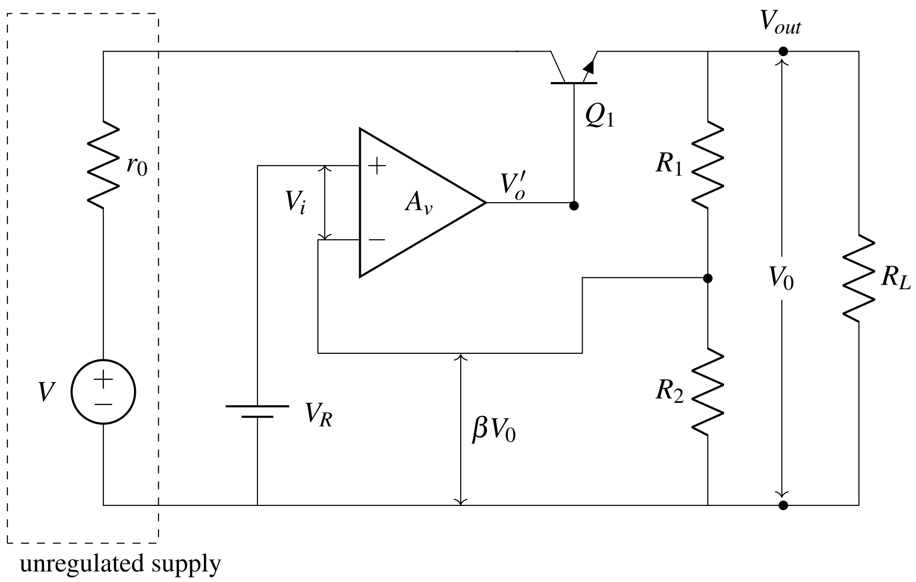


FIGURE I-A(1) Regulated power-supply system

If we assume that the voltage gain of the emitter follower $Q_1$ is approximately unity, then

$$V_0' \approx V_0$$

and

$$V_0' = A_V V_i = A_V (V_R - \beta V_0) \approx V_0$$

or

$$V_0 = \frac{V_R(A_V)}{1+\beta A_V} = V_R \left( \frac{1}{(\frac{1}{A_V} + \frac{R_2}{R_1+R_2})} \right)$$

Neglecting $\frac{1}{A_V}$ where $A_V$, the gain of the amplifier is $\approx 10^4$

$$V_o = V_R \left( 1 + \frac{R_1}{R_2} \right)$$

The output voltage can be changed by varying the feedback factor $\frac{R_1}{R_2}$. The emitter follower is used to provide the current gain because the current supplied by the amplifier may not be sufficient.

#### 1.5.1.1 Monolithic Voltage Regulators

Innumerable power-supply circuits have been designed based on general principles discussed above utilizing vacuum tubes, transistors and monolothic voltage regulators. These have a regulation as low as 0.001% and are capable of delivering several tens of amperes of current. The availability of low cost and high performance monolithic integrated circuits has made the design of power-supply considerably simpler. The small size and low cost has allowed greater flexibility in voltage levels and for regulation of individual stages thus providing improved isolation and decoupling of these stages.

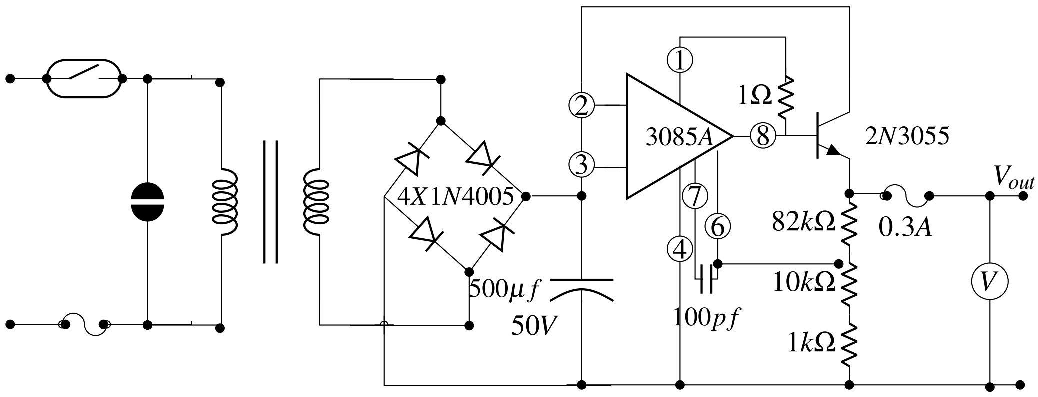


FIGURE I-A(2) 5V to 25V (0 to 0.5A) Regulated power-supply

Indigenously manufactured voltage regulator BEL 3085A has been used in the design of the present regulated power-supply. Some of its important characteristics are: reference voltage 1.6volt, d-c input 40 volt, current 100 mA and an overload protection. Thu, the unit is capable of delivering an output voltage continuously vriable from 3.0 Volt to 36 volt.

The complete circuit diagram of the regulated 5-25 volt 500 mA power-supply is shown in Fig. I-A(2). The current capability has been increased by use of a series-pass transistor 2N3055. The output voltage is monitored continuously by a d-c meter. The same meter can also measure the current drawn from the supply. A simple switch selects either of these modes. The supply has a regulation better than 0.2% for 20% line fluctuations and no load to full load current change.

References

BEL data sheets on voltage regulator 3085A and 3085B
RCA handbook of linear integrated circuits.

### 1.5.2 Appendix I-B THE METERS

#### 1.5.2.1 d-c MEASUREMENTS

All the meters on the boards are d-c current measuring instruments. They are used with suitable circuitry to measure d-c currents and voltages or a-c currents and voltages.

For measuring currents, an 80 ohms meter of range 0-1mA or 0.5-0-0.5 mA is used. For voltage measurements, a meter with a resistance of $2k\Omega$ and a range of 0-50 $\mu A$ and another, centrally pivoted, with a resistance of $2k\Omega$, and a range of 25-0-25 $\mu A$ are used.

For measuring currents larger than the specified range, the current meter is suitably shunted and calibrated. For example, suppose the 0-1 mA meter with 80 ohms resistance is to read a current of 100 mA to give full scale deflection. It is then necessary to arrange the shunt R [See Fig. 1-B (1)] to give 1 mA current through the meter. Calculation shows that R must be $80/90 \approx 0.8 ohm$.

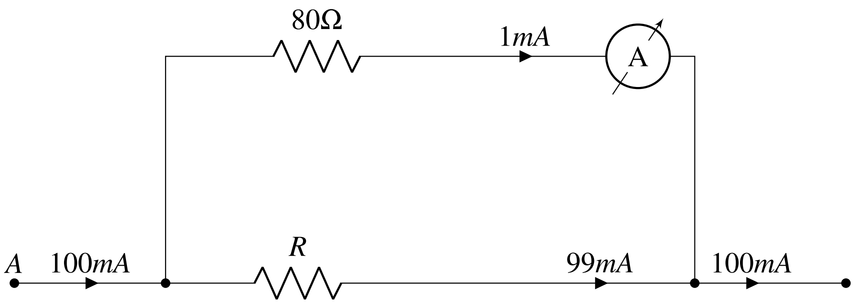


FIGURE I-B(1)

Now in order to have such a meter of multiple ranges all you have to do is to arrange a number of shunts as in Fig. I-B (2) so that you can just plug in at the input terminals AB and link the point P to the desired range.

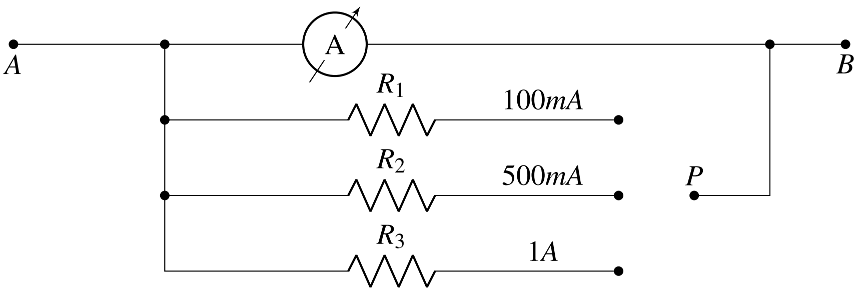


FIGURE I-B(2)

However, think what would happen if you connect the meter in a circuit carrying 1 ampere current without first linking P to the 1.0 A range. *The method is thus quite simple in principle but not safe in practice*. We have therefore, resorted to another arrangement of shunts called *Ayrton* shunting shown in Fig. 1-B (3). The shunts $S_1$, $S_2$ and $S_3$ etc. are all connected in series with each other and the whole series is then connected across the meter. You can plug in at A and any of the points B so as to work with the desired range. The values of shunts are so chosen that when you plug in at $B_3$ the total shunt value ($S_1$+$S_2$+$S_3$) across the meter is 0.8 ohm. If you plug in at $B_2$ the shunt ($S_1$+$S_2$) would be 0.16 ohm, just the value needed for converting the 80 ohm meter to measure a current upto 500 mA.

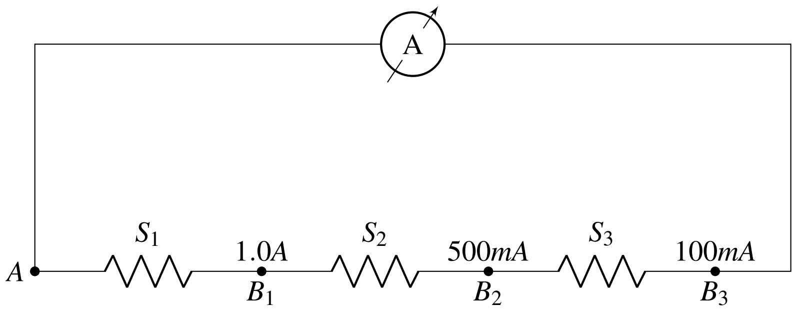


FIGURE I-B(3)

Similarly, when you plug in at $B_1$ for 1.0 A, the shunt $S_1$ would be 0.08 ohm.¶ Thus for the

¶In plugging in at $B_1$ the resistance $S_2+S_3$ adds up in series with the meter resistance 80 ohm. But the effect is meter of 80 ohms the desired ranges can be obtained by making $S_1$+=$_S$=0.08 ohm, and $S_3$=0.64 ohm. The meter with this shunting arrangement would never be without a shunt, no matter where you plug in and is therefore, relatively safer.

Various shunts are provided to give the specific ranges of measurement and you will find these ranges marked on the board. In choosing the proper range for your use, start with the highest one and work down until you have a comfortable range. In other words, *do not pass a high current through the meter* and damage it.

The principle of measuring d-c voltage may be understood with a simple illustration. Suppose the voltage you wish to measure is approximately 5 volts. It will be convenient to have the voltmeter so arranged that it gives full scale deflection for (say) 10 volts. Since our meter gives full scale deflection for $50\mu A$ current, one must connect a $200k\Omega$ resistor in series with the meter[Fig. I-B(4)]. This is simple Ohm's law : 10 $volt \div 50\mu A$= $200k\Omega$.

A similar calculation shows that this meter of $50\mu A$ range, for conversion to voltmeter, needs $2M\Omega$ resistance for 100 volt range, $20M\Omega$ resistance for 1000 volt range, and so on- in general $20k\Omega$ per volt range. This quantity 'y ohm per volt' is a useful measure for the quality of the voltmeter. The reason is that the voltmeter resistance has to be much greater that the resistance across which the voltage is to be measured. In Fig. I-B(4) this condition is satisfied ($200k\Omega \gg 1k\Omega$). But if the same voltage has existed across (say) a $100k\Omega$ resistor, this voltmeter would not be satisfactory.

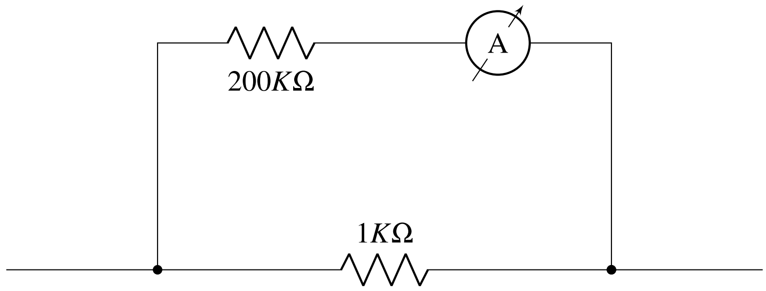


FIGURE I-B(4)

Note that 'y ohm per volt' really means that the basic meter draws $1/y$ amp current for full-scale deflection.

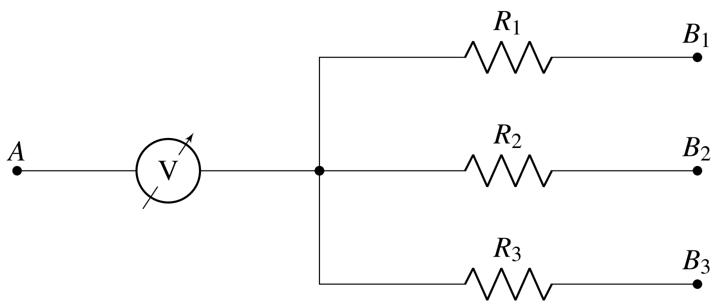


FIGURE I-B(5)

For multiple ranges the series resistors are arranged as shown in Fig I-B(5). All you have to do is to plug in at A and any of the points B for the desired range across any element of potential difference in the circuit. *Caution against overloading the meter should again be adhered to*.

negligible since $S_2 + S_3 \ll 80$ Ohm.

In the light of the above illustrations, try and understand the remarks at the beginning of this Appendix about all meters being current measuring instruments, in principle.

### 1.5.2.2 a-c MEASUREMENTS

These boards use the *same meters for a-c measurements* but obviously they cannot be used directly as for d-c. Why not? The frequency of the a-c mains which we usually work with is 50 hertz which means that the current reverses direction every $1/100$th sec. The time of response[‖] of the d-c meters used is of the order of $1/10$ sec. i.e. an order of magnitude greater than the reversing time for the a-c signal. This means that even if we employed a centrally pivoted meter to take care of the reversing of the current, its needle would not be able to keep track of the a-c signal and consequently would just remain stationary at the central position.

A simple arrangement for measuring a-c peak voltage is shown within the dashed box in Fig.I-B(6). Here r represents the small forward resistance of the diode D and other series effective resistance. To understand the working, assume that we have an a-c signal at 50 cycles; 20 volts peak across the terminals a, b. The value of C is such that rC is very much less than the time period of variation of the voltage i.e. $1/50$ sec.

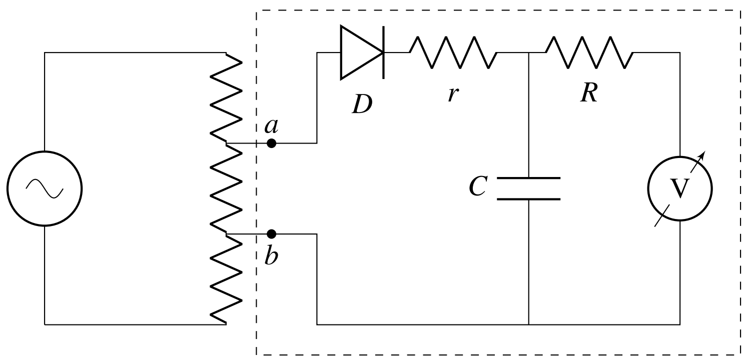


FIGURE I-B(6)

When a is rising up to the peak voltage 20 volt, the capacitor charges to this potential with the diode conducting. Then as a drops below this potential diode becomes non-conducting and the capacitor discharges (very slowly) through the meter V only. Thus the voltage across the capacitor follows the solid line in Fig. I-B(7)

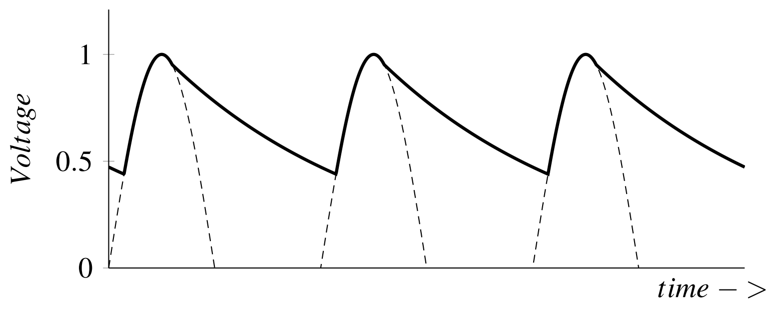


FIGURE I-B(7)

The resistance R in the discharge circuit (i.e. the resistance of the voltmeter plus the resistance in series to it) is kept sufficiently high so that the fall in the voltage across the capacitor between consecutive cycles is small. Obviously, the condition for this is

[‖]In a simple way the time of response of a meter may be understood to mean the time required for it to reach a full-scale deflection after the current is switched on.

$$RC \gg \frac{2\pi}{\omega}$$

The above arrangement, simple though it is, is unsuitable in many situations. Consider, for example, the circuit illustrated in Fig. I-B(8).

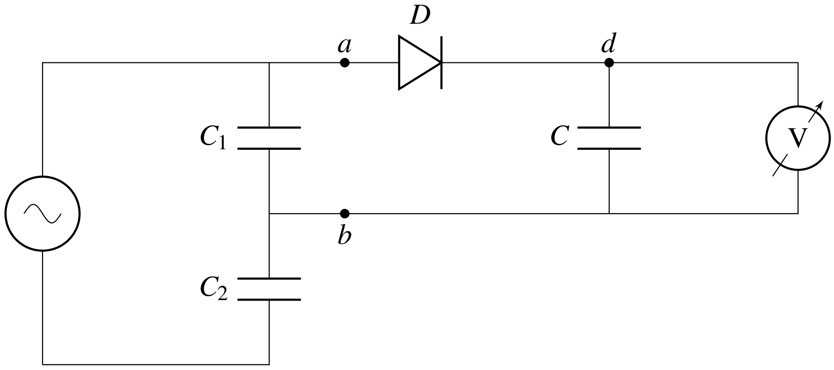


FIGURE I-B(8)

Here, our problem is to measure the a-c voltage across $C_1$, when a source is connected to $C_1$ and $C_2$ in series. When we connect our voltage measuring arrangement across $C_1$ the meter at first shows a deflection, but gradually retraces to zero. This is because of capacitor $C_2$, which gets charged through R and the meter (in a time $\approx C_2R$) to the peak potential of the source. Once this happens there is no current through the meter, because the potential at point d is the same as that at b and the diode does not conduct even during the positive cycle. The point is that the capacitor $C_2$ is unable to discharge through the reverse biased diode and thus potential at b stays at the positive peak value. The upper terminal of $C_2$ all the time goes from $-V_0$ to $V_0$ along the source.

To overcome this, we have introduced a bridge circuit for all a-c measurements.

### 1.5.2.3 THE BRIDGE CIRCUIT

The bridge circuit involves four diodes connected as shown in Fig.I-B(9), along with C, R and meter V. The voltage to be measured is across PQ and is applied to the bridge terminals ac.

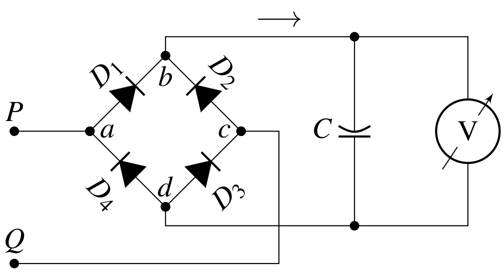


FIGURE I-B(9)

During the half of the cycle when P is positive, the diodes $D_1$ and $D_3$ conduct so that current flows along Pab and then to dcQ along the direction of the arrow in Fig. I-B (9).

In the other half of the cycle $D_2$ and $D_4$ conduct so that the current flows along Qcb and then daP, again along the direction of the arrow. The capacitor C is charged to the peak value, twice in each cycles, then discharge through the resistor R between half cycles being quite small. ln Fig 1-B(10) this is shown the upper curve is the applied voltage and the lower one the voltage across the capacitor [compare with Fig. I-B(7)]. The meter is now calibrated to read the appropriate rms value of voltage (rather than the peak value).

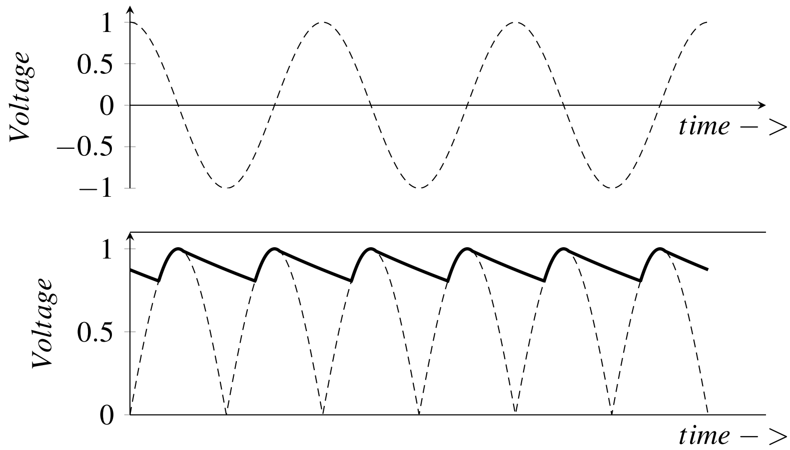


FIGURE I-B(10)

With reference to Fig. I-B(8) we had discussed how the point Q would not be able to lose its positive potential once acquired if the element between P and Q were a capacitor and we used a single diode. The bridge circuit avoids this by providing path Qcb RV daP, which is the same as the discharging path except for a different pair of diodes.

# CHAPTER 2

# 2. Charging and Discharging of a Capacitor

## 2.1 Capacitors

A system of charges, physically separated, has potential energy. The simplest example is that of two metal plates of large area carrying opposite charges so that the potential difference is V.

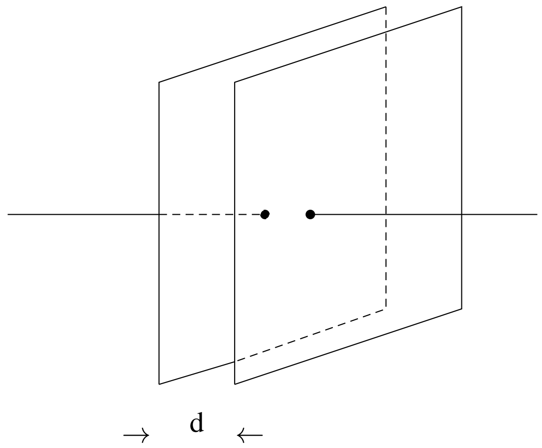



FIGURE 2.1

The energy stored is $\frac{1}{2}CV^2$ where C is the capacitance of the system. C is defined as the charge (on either plate) per unit potential difference and depends essentially on the geometry of the system. In the above case the capacitance is given by

$$C = \varepsilon_o \frac{A}{d} \tag{2.1}$$

in mks units, where $A$ is the area (in $meter^2$), $d$ is the separation (in meters), $\varepsilon_o$ is a constant $(8.85 * 10^{-12})$ farad/meter and the unit of capacitance is a farad. (Refer to any standard text for the derivation of this formula).

A system, such as the above one, is called a condenser or, in modern parlance, simply a capacitor. We shall adopt the modern usage.

It must not be assumed that a capacitor is always a set of plane parallel plates. Many other geometrical arrangements may be used and often are more practical ( See Appendix II-A).

## 2.2 RC Circuit

The energy may be delivered by a source to a capacitor or the stored energy in a capacitor may be released in an electrical network and delivered to a load. For example, look at the circuit in Fig.2.2. If you turn the switch $S_1$ on,

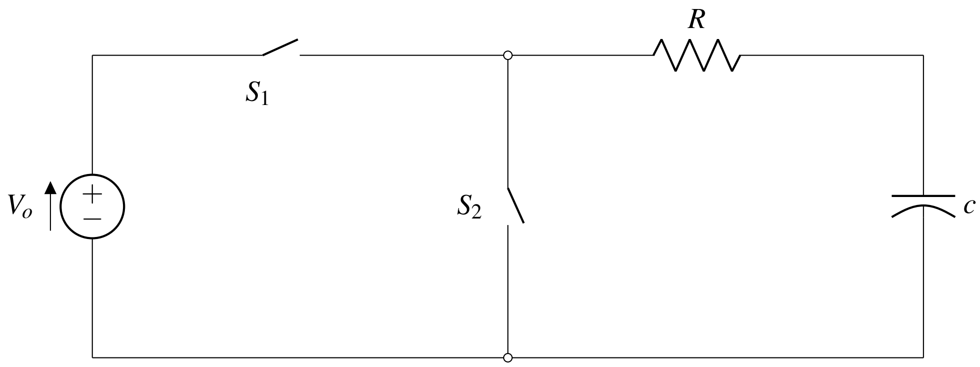


FIGURE 2.2

the capacitor gets charged and when you turn on the switch $S_2$ ($S_1$ off) the capacitor gets discharged through the load. The rate at which energy is delivered in either case will be controlled by the rate at which the charge moves, i.e. the current; this, of course, will depend on the resistance offered. It will be seen, therefore, that the rate of energy transfer will depend on RC where C is the capacitance and R some effective resistance in the circuit. It can be shown (Appendix II-B) that the charging of a capacitor can be represented by the relation

$$q = q_o(1 - e^{-t/RC} \tag{2.2}$$

where q is the charge on the plates at time t; similarly,the discharge occurs according to the relation

$$q = q_o e^{-t/RC} \tag{2.3}$$

Thus, the rate at which the charge or discharge occurs depends on the "RC" of the circuit. The exponential nature of the charging and discharging processes of a capacitor is obvious from Equations (2.2) and (2.3). You will have ample opportunity to learn more about it through the experiments that follow. From Equation (2.3) it can be seen that RC is the time during which the charge on the capacitor drops to 1/e of the initial value. Further, since RC has dimensions of time, it is called the *time constant* of the circuit.

In the following series of experiments, you will study the time variation of charge, voltage and energy in RC circuit.

## 2.3 The Network Board - 2

The network board for these experiments consists of a number of resistors and capacitors and two d-c meters. The centrally pivoted meters facilitate measurements during both charging and discharging of a capacitor. Fig.2.3 shows the scheme of arrangement of these on the board and their connection underneath it.

The capacitors are of electrolytic type (since you need high values of capacitance). These are meant for use with d-c power and great care must be taken to connect them with the right polarity.

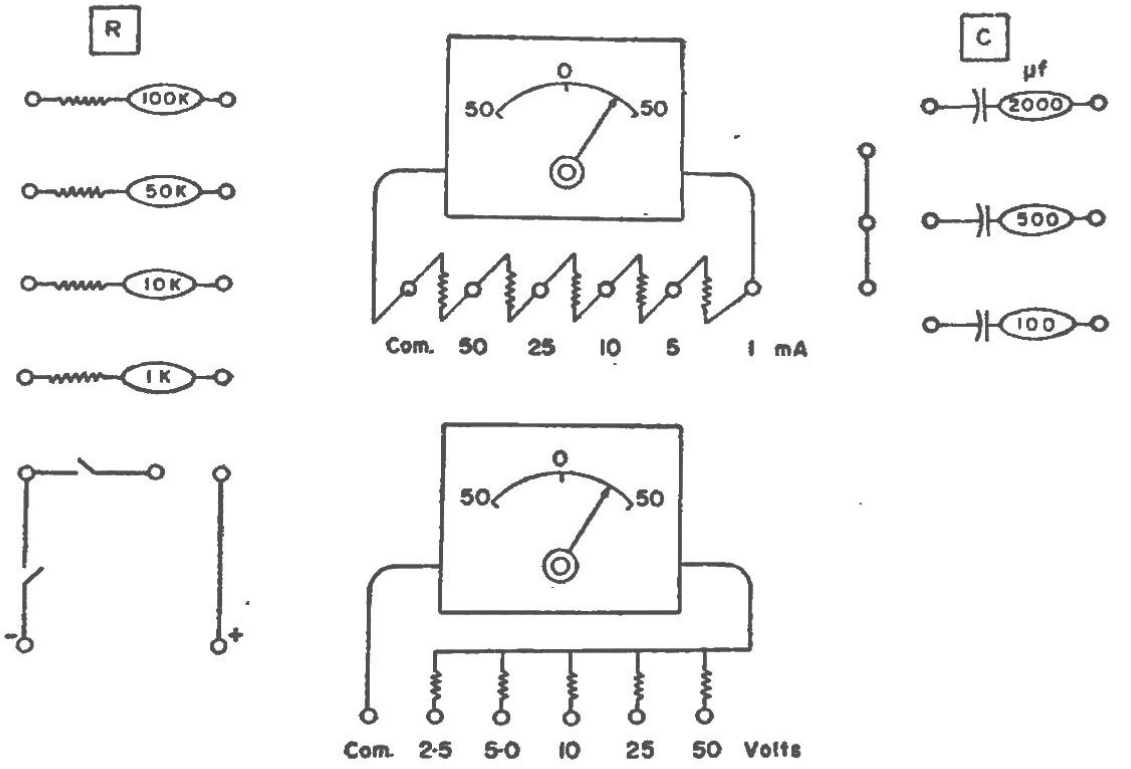


FIGURE 2.3

In order to make time constant RC of the circuit large the resistors also need to have high value and are, therefore, carbon film type(See Appendix II-C). Remember values marked on both R and C are not absolutely dependable. The resistance values are given within $(\pm 1\%)$ but the capacitance values have a tolerance of $(\pm 10\%)$ or more.

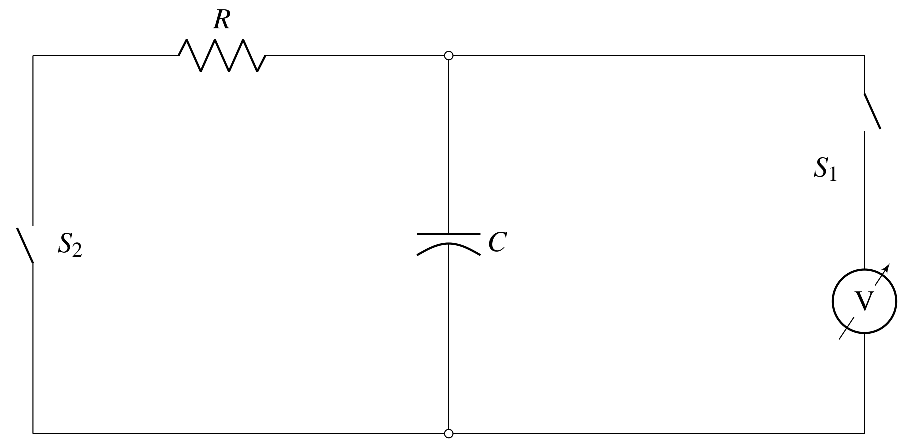


FIGURE 2.4

A regulated d-c power -supply and a timer (metronome) are also provided along with the board. The power-supply has already been discussed (Appendix I-A). Use of higher voltages (20 to 25 volt) from this supply would enable you to get higher resistance of the voltmeter and thus reduce the unwanted discharge through the voltmeter and considerably improve the performance of the experiments. This would be obvious from the following discussion. If you look at Fig.2.4 relating to the discharging of a capacitor, you would realize that on turning the switches $S_2$ and $S_3$ on, the capacitor would discharge through both the load R and the voltmeter V. If $R_V$ be the resistance of the voltmeter, the effective leakage resistance $R'$ would be given by

$$R' = R\frac{R_V}{R+R_V}$$

The unwanted discharge through the meter can, therefore, be reduced only by making $R_V$ much higher that R. This is accomplished in a simple way by using higher voltage source and employing a higher range of the meter for detection. However, even this would not be adequate in case of smaller C values where you should employ a sort of 'sampling method' for voltage measurements. This consists in turning on the switch $S_3$ only at the instant when a measurement is to be made.

The metronome gives audible 'ticks' at regular time intervals (Appendix II-D) and may be used with advantage in place of a stop watch. The interval can be adjusted between 0.5 sec. to 10 sec. In taking meter readings at these intervals, you may find it difficult to read the meter, say every 2 seconds or so. In that case, take one set of readings at 0,6,12,18,..... sec., then the next set at 2,8,14,20,.....sec. and so on until you have a complete set of readings every 2 sec.

## 2.4 Experiments

### 2.4.1 Charging of a capacitor

#### Experiment II-A: To study the charging of a capacitor in an RC circuit.

Take a resistor and a capacitor and complete the circuit as shown. Switch on the metronome and the circuit simultaneously.

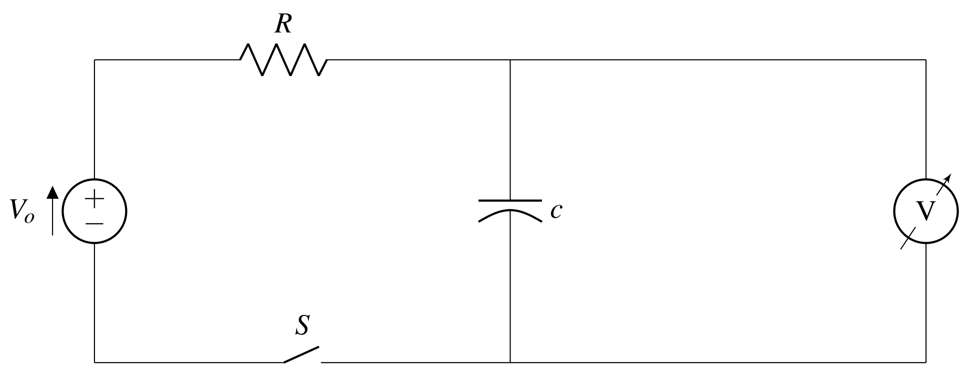


FIGURE 2.5

Read the voltmeter every $n^{th}$ tick, until the voltmeter indicates a maximum value $V_0$ *. Plot the voltage V across the capacitor as a function of time.[Fig.2.6(a)].

*Theoretically speaking, in the case of a pure capacitor, the voltage across it should become equal to the source voltage $V_o$ when the capacitor is fully charged. In practice, it is very seldom so. This is because there is always a leakage of charge across the capacitor

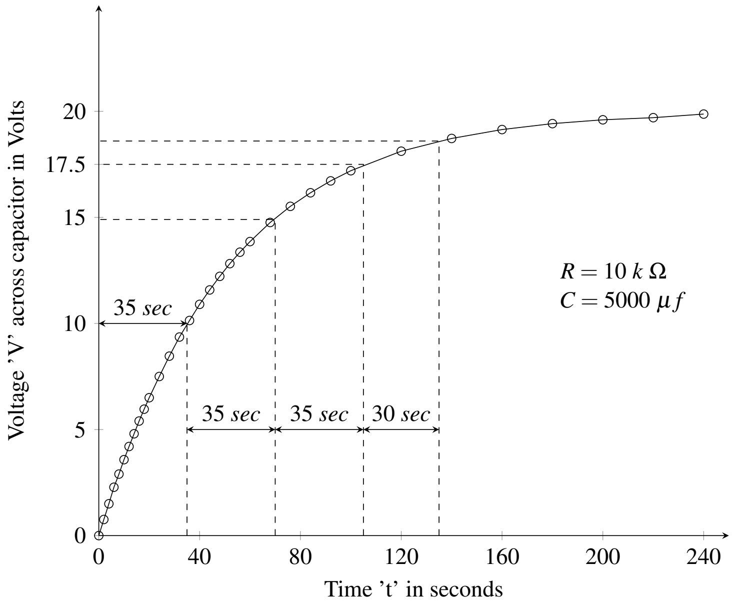


FIGURE 2.6(a)

To analyze the results one procedure is as follows. On the voltage scale in Fig.2.6(a) mark points corresponding to

$$V_o - V = 1/2\ V_0, 1/4\ V_0, 1/8/\ V_0, 1/16\ V_0, ..(1/2)^n\ V_o$$

Then from the graph determine the corresponding times. You would find that the *intervals* between successive steps come out equal within experimental limits. This kind of rise is called an *exponential* rise of V; it really means also that $V_o - V$ falls exponentially. In equal times $V_o - V$ falls by equal *factors*. You could take the factor as 1/2 or 1/3 or 1/e or 1/10; in each case the time intervals come out equal.

Let the interval for $V_0 - V$ to fall by a factor 1/2 at each step be called $T_{1/2}$. Then the experimental result can be put as

$$V_o - V = V_o(1/2)^{t/T_{1/2}}$$

which can be written as

$$V = V_o[1 - 2^{-t/T_{1/2}}] = V_o[1 - e^{-0.69/T_{1/2}}] \tag{2.4}$$

since $log_e 2 = 0.69$. In the case of Fig.2.6(a) we find $T_{1/2} = 35$ sec.

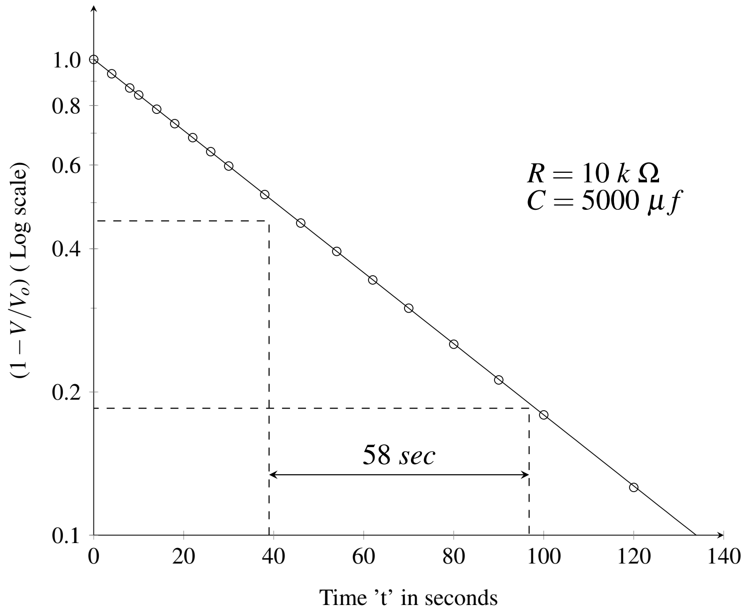


FIGURE 2.6(b)

Alternatively, you could plot a graph of $log(V_o - V)/V_o$ against time t [Fig.2.6(b)]. If this graph is a straight line, it means that Equation (2.4) holds, i.e., the potential rise is logarithmic. Also the slope of the line gives $-0.69/T_{1/2}$, from which you deduce $T_{1/2}$.

According to theory (See Appendix II-B) the voltage across the capacitor during the charging is given by

$$V = V_o(1 - e^{-t/RC}) \tag{2.5}$$

Thus $T_{1/2}$ is related with RC through

$$T_{1/2} = 0.69RC \tag{2.6}$$

RC is called *rise time* of the circuit. You may compute RC from the nominal values marked on the components and compare this RC with $(1/0.69)T_{1/2}$. The agreement may not be good, because the rated values of R and C (particularly the latter) could be off by as large as 10%. The value determined by you is probably a more reliable value than the one computed from the specified R and C.

### 2.4.2 Discharging of a capacitor

#### Experiment II-B: To study the discharging of a capacitor.

As shown in Appendix II-B, the voltage across the capacitor during the discharge can be represented by

$$V = V_o e^{-t/RC}$$

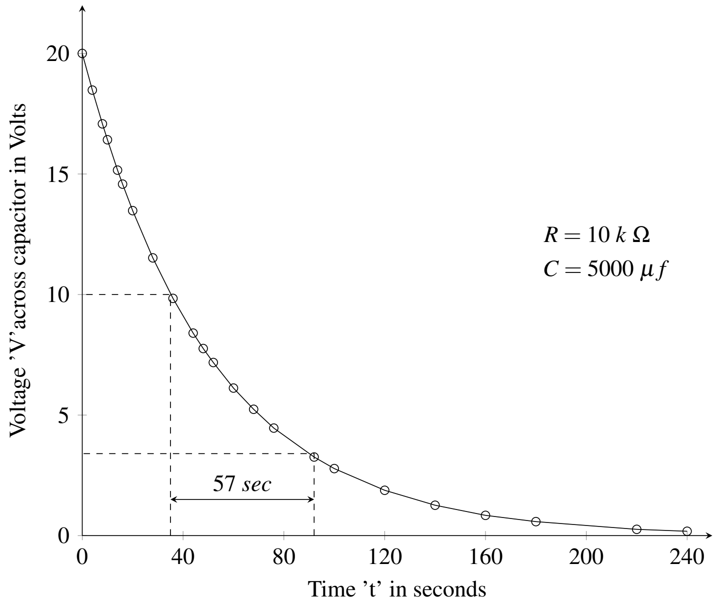


FIGURE 2.6(c)

You may study this case exactly in the same way as the charging in Experiment 2-A. However, remember that for the case of discharge $\frac{V_o - V}{V_0}$ has to be replaced by $\frac{V}{V_0}$ and $log\ (V_0 - V)$ by $log\ V$. (Why?). You would find that for the same set of R and C the time $T_{1/2}$ has the same value as in Experiment 2-A. Figs. 2.6(c) and 2.6(d) show the typical results graphically.

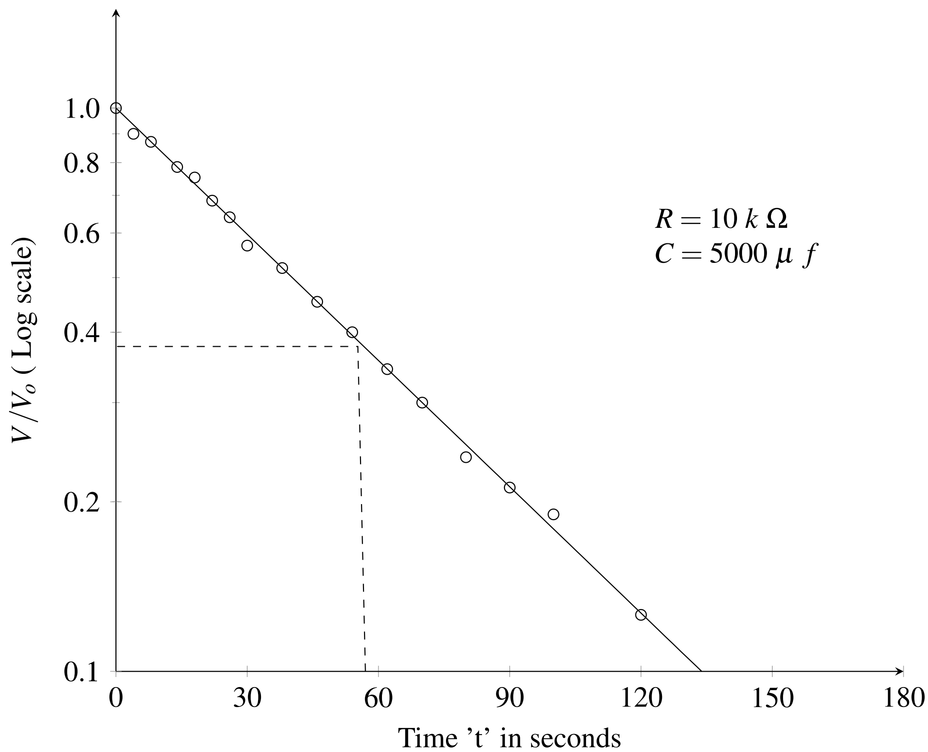


FIGURE 2.6(d)

In the circuit shown (Fig.2.7), if the switch is turned on at time $t = 0$ and turned off at instant $t = t_1$, the voltage across input terminals AB ideally behaves as in Fig.2.8. Plot quantitatively the output across PQ in view of study in Experiments 2-A and 2-B. You can think of the RC combination as a 'box' with input terminals AB and output terminals PQ. Suppose the circuit in the above question had been on for some time before the switch was suddenly disconnected . Display the *input* voltage as a function of time.

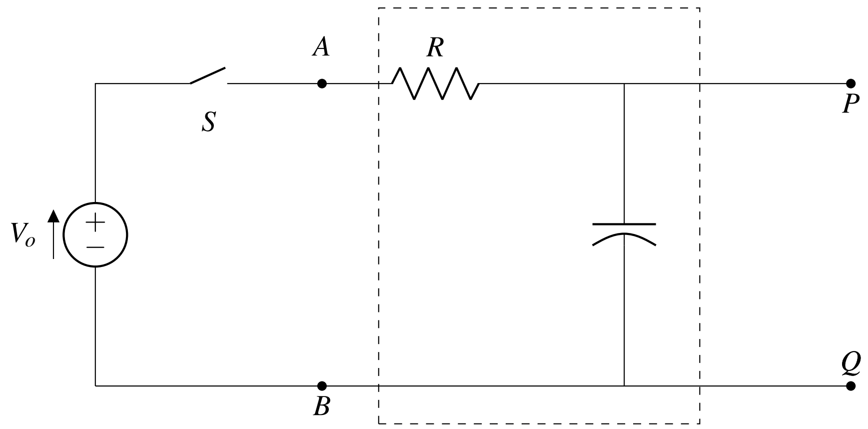


FIGURE 2.7

Also display the *output* voltage also as function of time.

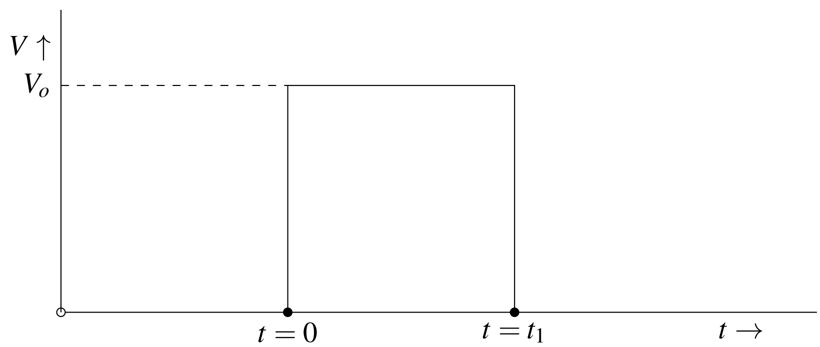


FIGURE 2.8

Next, assume that the 'box' is now wired as in Fig.2.9, with PQ across the resistor R.

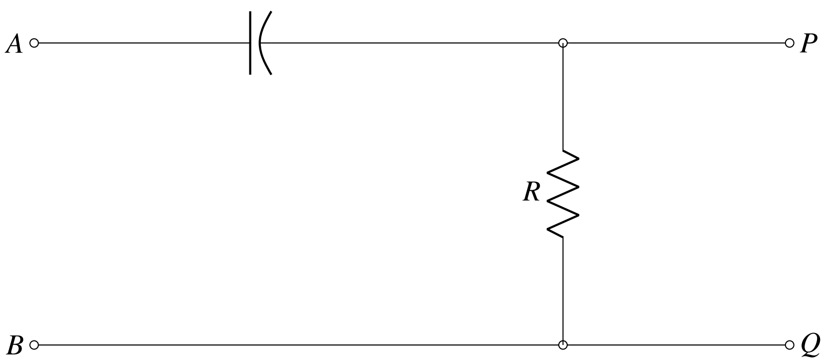


FIGURE 2.9

Discuss the input and output when the switch (not shown) is turned on and, later, turned off.

### 2.4.3 Current during charging and discharging

**Experiment II-C: To study the current flow during charging and discharging of a capacitor.**

The current flowing through an RC circuit is given by (Appendix II-B)

$$I = I_o e^{-t/RC} \tag{2.7}$$

for the charging circuit and

$$I = -I_o e^{-t/RC} \tag{2.8}$$

for the discharging circuit. Thus the current follows the same behaviour with time in both cases except that the direction of the discharge current is opposite to that of the charging current.

Connecting the milliammeter in series with the resistor and the capacitor(Fig.2.10), study the behaviour of the current in the two cases(Fig.2.11). Pay particular attention to the reversing of the current in the circuit. This is why a centrally pivoted current meter is provided.

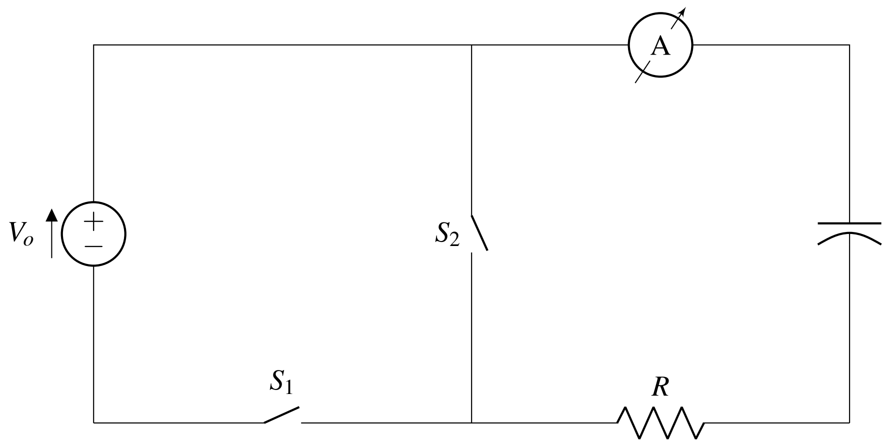


FIGURE 2.10

Also, if you connect the voltmeter across R, in addition to the reversing of polarity in the voltage across R, you would discover that the whole of voltage appears across it when you commence the charging or the discharging.

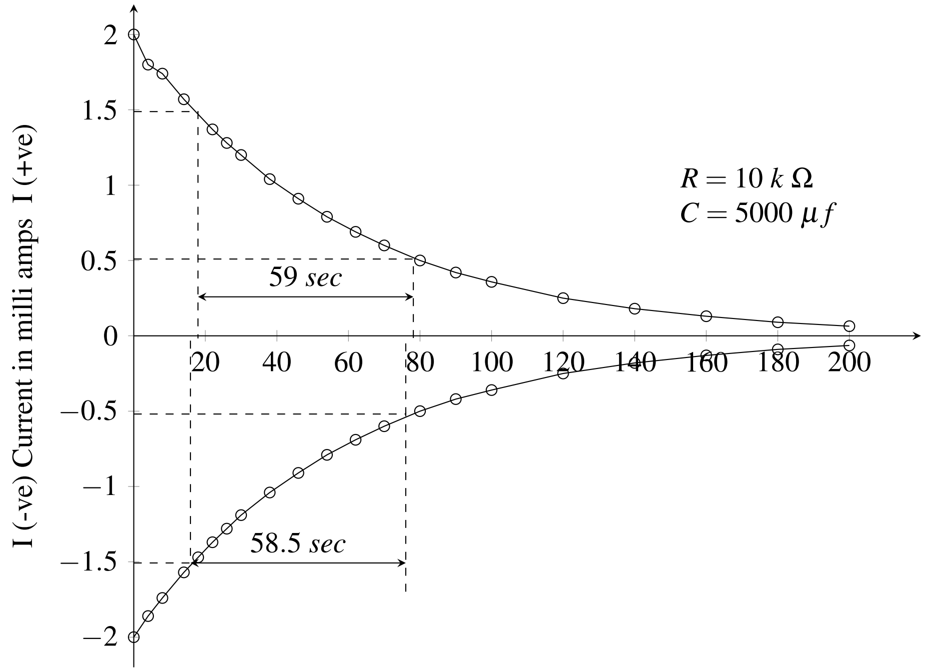


FIGURE 2.11

Also verify if the maximum current $I_0$ at the commencement of the charging and the discharging is given by

$$I_o = \frac{V_o}{R} \tag{2.9}$$

Further, you can see that at all instants of time

$$I = \frac{V_R}{R} \tag{2.10}$$

**Exercises Pertaining to Experiments 2-A, 2-B and 2-C**

a) Change the voltage $V_o$ of the power-supply and see if for a given RC,the time $T_1/2$ remains the same.Do you expect it to change ?.
b) For a known resistance, the time $T_1/2$ determines the capacitance. Use this to determine first $C_1$, then $C_2$ and finally the effective capacitance C with both $C_1$ and $C_2$ in parallel (Fig.2.12).

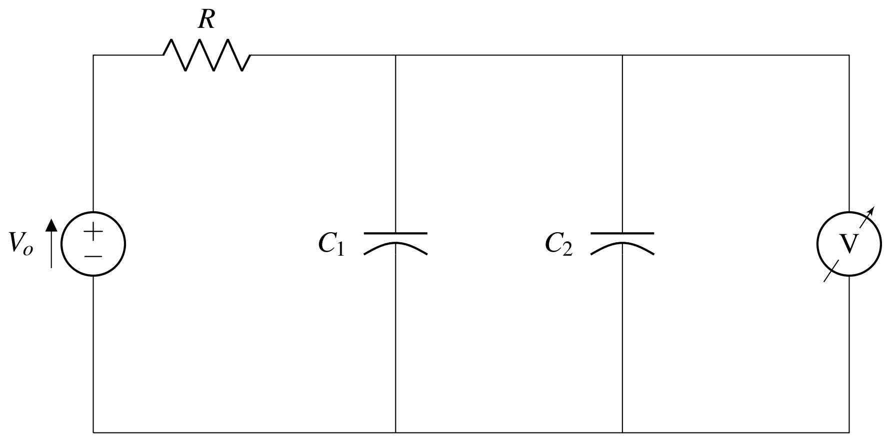


FIGURE 2.12

Verify the law $C = C_1 + C_2$ where $C$ is the effective capacitance of the combination in parallel. Try this with various resistors R.

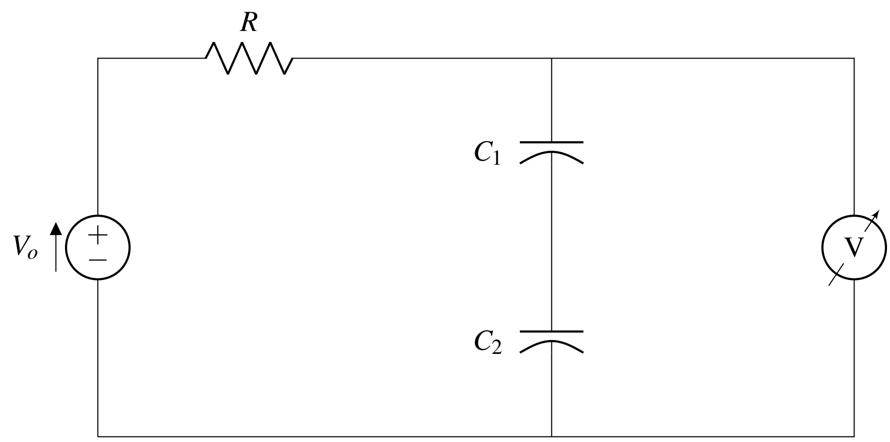


FIGURE 2.13

c) Use exactly the same method ( by measuring $T_1/2$) to verify the law

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \tag{2.11}$$

for a set of capacitors in series (Fig.2.13). Try with different values of R as well.

d) Charge a set of capacitors connected in series.(Roughly, about 5 times $T_1/2$ will charge the capacitors close enough to the maximum voltage). Measure the voltage across each and establish the law

$$C_1V_1 = C_2V_2 = C_3V_3 \tag{2.12}$$

e) Connect a set of capacitors in parallel. Measure the current through each of them† after fixed interval of time either during the charging or during the discharging operation and establish the relation :

$$\frac{C_1}{I_1} = \frac{C_2}{I_2} = \frac{C_3}{I_3} \tag{2.13}$$

(In verifying such relations as 2.11, 2.12 and 2.13, make sure to measure the capacitance yourself and not just trust the rated values).

f) With an RC time of around 30 sec.,measure the voltage across R as a function of time while charging and discharging the capacitor. Pay particular attention to the polarity of voltage cross R in each case. *It is for this reason that the voltmeter provided is a centrally pivoted one*. You would also notice that in both cases with the passage of time the voltage across the resistor goes on falling until it becomes zero when capacitor is fully charged or discharged.

If you use two voltmeters and measure the voltages across R and C simultaneously you can also verify that at all instants of time

$$V_R + V_C = V_0 \tag{2.14a}$$

during charging, and

$$V_R + V_C = V_0 \tag{2.14b}$$

†You will have to use the terminals provided at the left of the capacitor (Fig.2.3) for connecting the current meter in series with the capacitors individually.

during the discharge. This is the verification of Kirchoff's law. Figures 2.14(a) and 2.14(b) show typical results.

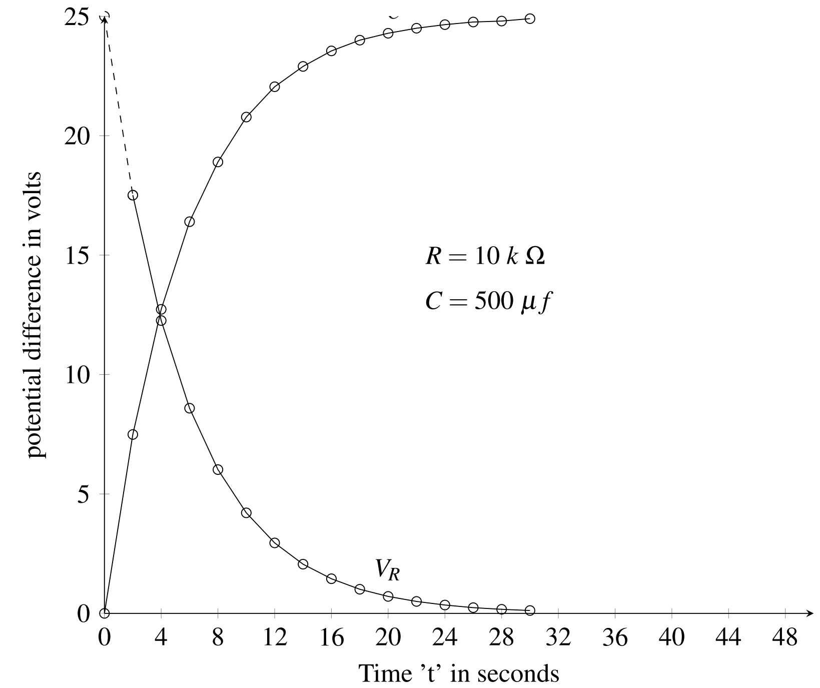


FIGURE 2.14(a)

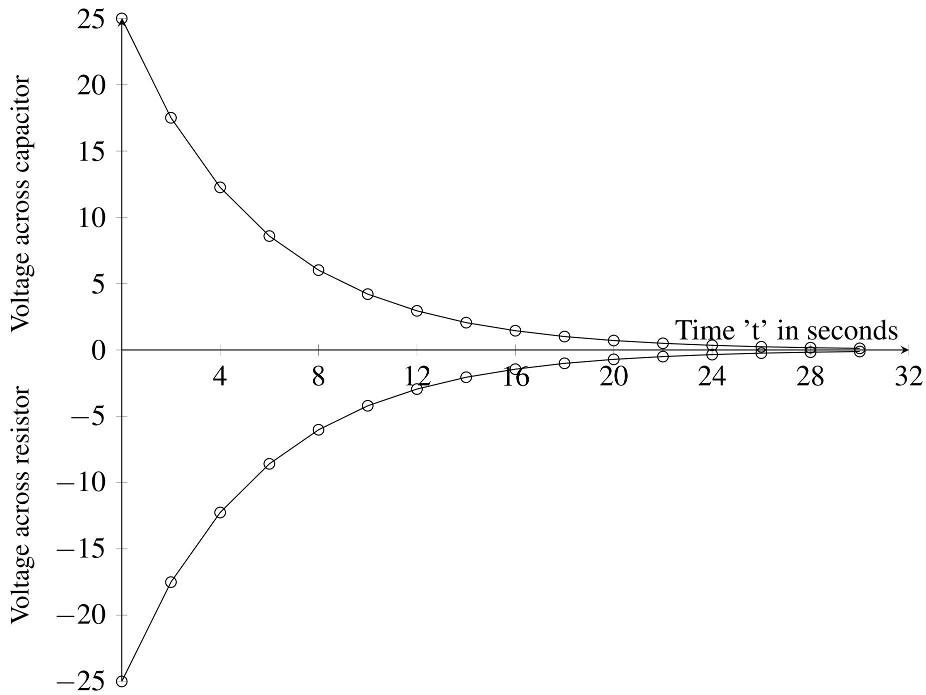


FIGURE 2.14(b)

### 2.4.4 Leakage resistance of a capacitor

**Experiment II-D: To estimate the leakage resistance of a given capacitor.**

Capacitors, once charged, do not maintain their charges indefinitely even when their terminals are disconnected. (But, they often maintain it for long times, so do not poke your fingers at these terminals. You are always advised to deliberately discharge the capacitor before leaving your experiment). A capacitor loses its charge by leakage either through the dielectric between the electrodes or through the insulators outside, which hold the capacitor electrodes in place. Thus, strictly speaking, any capacitor may be effectively represented as in Fig.2.15 where $R_c$ represents the leakage resistance of the capacitor C. It is of order of a few megohm.

In the case of an ideal capacitor ($R_c = \infty$) when fully charged, the voltage $V_C$ across it should be equal to $V_0$(Fig.2.5) and the final value of the charging current I in the circuit(Fig.2.11) should be zero.

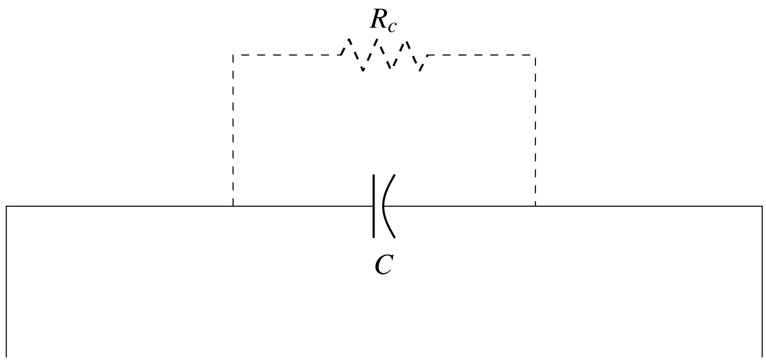


FIGURE 2.15

In practice, as you would discover during the course of these experiments, this is not the case. $V_C$ is always less than $V_o$ and charging current never drops down to zero. It is easy to understand these facts if you remember the true representation of a capacitor (Fig.2.15).

Take a resistor R and a capacitor C so that the time constant RC is of the order of 10 sec. or more. Connect them in series with a milliammeter [Fig.2.16(a)] (Note how the capacitor has been represented). Turn on the switch and confine your attention to the current meter to observe how the charging current drops with time. After a time (5 RC or more) the capacitor is expected to be fully charged and the current drops to zero.

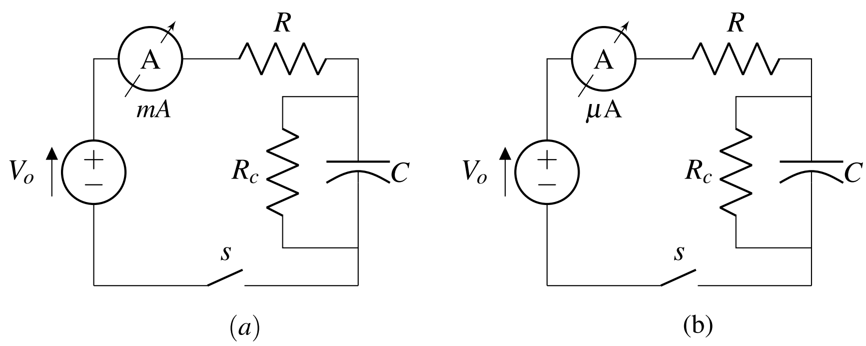


FIGURE 2.16

On your meter it may indeed appear as if the current has become zero but if you replace the milliammeter by a microammeter [‡] of movement $50\mu A$ or less[Fig.2.16(b)] you would find a small

‡You may use the $25-0-25\mu A$ meter i.e. the voltmeter on the Network Board -2 for this purpose. You will have

and steady current flowing persistently no matter how long you wait. Measure this leakage current $I_c$. Assuming voltage te capacitor to be same as $V_0$ , the supply voltage (this is not quite correct), calculate the order of leakage resistance $R_c$ by

$$R_c = \frac{V_o}{I_c} \tag{2.15}$$

You must learn to make approximations like these ($V_C \approx V_o$) and understand why such approximations do not matter when it is only the order of magnitude of a quantity you are interested in. You should further be able to appreciate the difficulty in measurement of $V_C$ with a meter of finite resistance and hence the importance of the approximation $V_C = V_0$. However, if you are interested in knowing the leakage resistance more precisely you may calculate it as follows:

$$R_c = \frac{V_0}{I_c} - R \tag{2.16}$$

Do you see how the approximation involved in Equation(2.15) is taken care of in Equation (2.16)?

If a capacitor of $100\mu$ f has a leakage resistance of 5 megohm. A voltmeter of 500 kilohms is connected across it to read the voltage. How much time would it take for the voltage to fall to a value $1/e$ times the initial value? Compare it with the case when the voltmeter resistance is infinite.

### 2.4.5 Energy dissipated during charge and discharge

#### Experiment II-E: To measure the energy dissipated in charging a capacitor.

Some energy is spent by the source in charging a capacitor. A part of it is dissipated in the circuit and the remaining energy is stored up in the capacitor. In this experiment we shall try to measure these energies.

With fixed values of C and R measure the current I as a function of time. The energy dissipated in time dt is given by $I^2Rdt$. The total energy dissipated is given by

$$E = \int I^2 R\, dt = R \int I^2\, dt \tag{2.17}$$

This integral can be evaluated by the graphical method as follows:

From the observed values of I plot $I^2$ versus t (Fig.2.17). The area under the curve gives the value of the integral and R times this area is therefore a measure of the energy dissipated in the circuit.

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to reach directly the terminals of the meter underneath the Board in order to see it as a $25\mu A$ current meter.

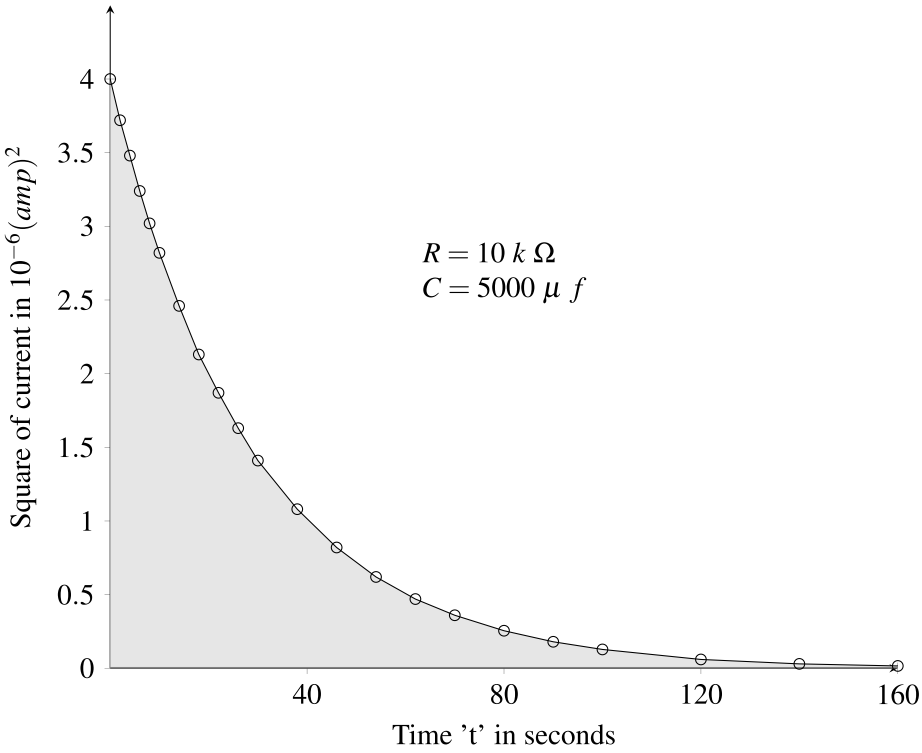


FIGURE 2.17

Does the energy dissipated depend on the value of the value of resistance?. A cursory glance at Equation (2.17) would indicate that it should. You can test this as follows:

Plot the $I^2$ , t curves for different values of the resistance R in the circuit and measure the area in each case. You will discover an amazing result- the energy dissipated thus would turn out to be independent of the charging resistance.

Repeating this experiment for the case of discharging, you will find that again an equal amount of energy is dissipated in the circuit. Since this energy in the case of discharging comes from the capacitor you can draw a very simple conclusion from these experiments. Of the total energy drawn from the source in charging of a capacitor, half is dissipated in the circuit and half is stored up in the capacitor irrespective of the value of the resistance. In other words, of the total energy spent in charging a capacitor you can recover only half of it.

In charging or discharging a capacitor through a resistor an energy equal to $\frac{1}{2}CV^2$ is dissipated in the circuit and is independent of the resistance in the circuit. Can you devise an experiment to measure it calometrically?. Try to work out the values of R and C that you would have to employ in this experiment. Remember, capacitors having high value of capacitance cannot withstand voltages higher than 50 to 60 vols and those which can withstand higher voltages have low values of capacitance.

Suppose the total resistance in the circuit including that of connecting wires is made zero, in what part of the circuit would the energy $\frac{1}{2}CV^2$ be dissipated now?. How would you modify your above calometric measurement for this case?.

## 2.4.6 Dependence of dissipation on C and V

### Experiment II-F: To study the dependence of the energy dissipated on C and V.

For a fixed voltage $V_0$, the energy dissipated is proportional to the value of C i.e. if $E_1$, $E_2$ etc. are the energies dissipated for capacitors $C_1$, $C_2$ etc. we shall have

$$\frac{E_1}{C_1} = \frac{E_2}{C_2} = \ldots \tag{2.18}$$

Measure the energies $E_1$, $E_2$ etc. graphically (Experiment 2-E) and check this.

For a fixed capacitance C, estimate similarly the energy dissipated for different values of the supply voltage $V_1$, $V_2$ etc. You may vary V from 5 to 20 volts or so. Establish the relation

$$\frac{E_1}{V_1^2} = \frac{E_2}{V_2^2} = \dots \tag{2.19}$$

In fact, the energy dissipated for different supply voltages is $\frac{1}{2}CV_2$ (Appendix II-B); see if you can verify this in all experiments discussed.

The result that the energy dissipated $\frac{1}{2}CV^2$ in an RC circuit is independent of R seems strange. Try and see if you can present an argument to justify this. Discuss this in the limiting cases $R \to 0$ and $R \to \infty$ also.

### 2.4.7 Adiabatic charging of a capacitor

#### Experiment II-G: To study the adiabatic charging of a capacitor.

Is there no way of eliminating or reducing the dissipation of energy $\frac{1}{2}CV^2$ in charging of a capacitor? The answer is yes, there is a way. Instead of charging a capacitor to the maximum voltage $V_0$ in a single step by using a source voltage of $V_0$ right from the start, if you charge it to this voltage by increasing the source voltage in small steps the dissipation of energy can be reduced. Theoretically speaking, if successive steps are infinitesimally small the dissipation can be entirely eliminated. This is called *adiabatic charging* of a capacitor. You can verify this with the following experiment.

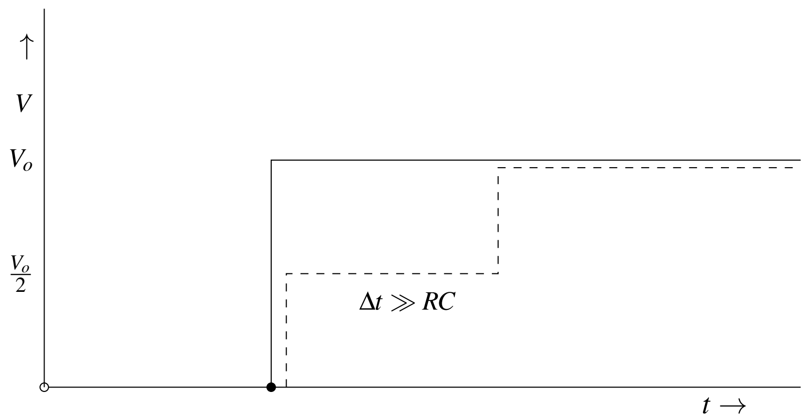


FIGURE 2.18

Suppose you want to charge a capacitor C to a voltage $V_0$. If you do that in a single step you know (Experiment 2-E) that an energy $\frac{1}{2}CV_0^2$ would have to be dissipated in the circuit. On the other hand, if you first charge the capacitor to a voltage $V_0/2$ and then from $V_0/2$ to $V_0$ the total energy dissipated would be $\frac{1}{4}CV_0^2$. (Why?) You can check this experimentally. The trick is to first keep the charging voltage to $V_0/2$, let the capacitor charge for a time much greater than RC of the circuit, disconnect the power-supply, increase its voltage to $V_0$. Plot $I^2$, t curves for the two parts and find out the total energy dissipated in the process. Compare this with the area of the curve obtained when the capacitor is charged to $V_0$ in a single step and you would find the former to be roughly one-half the area in the latter case. The charging voltage in the two cases can be represented as shown in Fig.2.18. Now think how you can reduce this loss further. Check your answer experimentally.

A capacitor of 1000 $\mu$ f is connected in series with a resistor of 2 kilohms. Calculate the energy dissipated in charging it to 20 volts in single step. How many equal steps will you have to employ to cut down this loss to one-tenth of its value.?. Show these steps graphically(as in Fig.2.18) taking care to mark the appropriate values of $\triangle t$.

Can you now think of an ideal charging method to reduce this loss to zero? Would it be possible to accomplish this in practice?

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# 2.5 APPENDIX

## 2.5.1 Appendix II-A CAPACITORS

### 2.5.1.1 Paper and Other Capacitors

Commercially available capacitors come in various forms for use in simple networks. A common one is the paper capacitor in which a pair of metal foils sandwich a thin paper. The whole assembly is then rolled into a bundle, dipped in wax and sealed against moisture. There may still be some leakage of charge through the paper particularly if the applied voltage is large. A practical consideration for a capacitor is always the voltage it can withstand without breakdown.

The capacitance of the system is somewhat increased when there is a dielectric (such as paper) between the electrodes. Other dielectrics commonly used are mica, ceramics and sometimes plastic films.

It can be seen that by reducing the distance between the electrodes one can increase the capacitance; but one cannot do this indefinitely. For a given voltage, electrical breakdown (i.e. current through the dielectric) occurs if the distance is too small. For example, if air is the dielectric and the capacitor is to withstand 100 volts, a separation of at least 1/10 mm is required. The capacitance of a parallel plate capacitor is given by

$$C = \varepsilon_o \frac{A}{d}, \varepsilon_o = 8.85 \times 10^{-12} farad/metre$$

One can see from this relation (the reader is advised to do this arithmetic) that no more than about 10 pico-farad per sq. cm (1 pico-farad =10$^{(}-12)$ farad can be achieved.

### 2.5.1.2 Electrolytic Capacitors

Some metals, like aluminium, when placed in a suitable electrolyte and made the positive electrode (i.e. aluminium is the positive electrode) form a thin film (about $10^{-6}$ cm) of oxide. This film has a very high resistance to a flow of current in one direction (from aluminium towards electrode) and a very low one in the reverse direction. Thus, provided we use the aluminium side as the positive one, we can obtain fairly large capacitance, a micro-farad per 10 $cm^2$ area with this kind of system when the aluminium and the electrolyte form the two electrodes. Even smaller film thicknesses can be made so that electrolytic capacitors can achieve as high as $10^{-4}$ farad for 10 $cm^2$. It is obvious that we cannot use an electrolytic capacitor with a-c unless we ensure that its polarity would not change. Other limitations are that they have a larger leakage current than the ordinary capacitors, their life is shorter, their capacitance may change somewhat after a few months (even the values marked on the new ones may vary by as much as 20%) and the working voltages for these are lower.

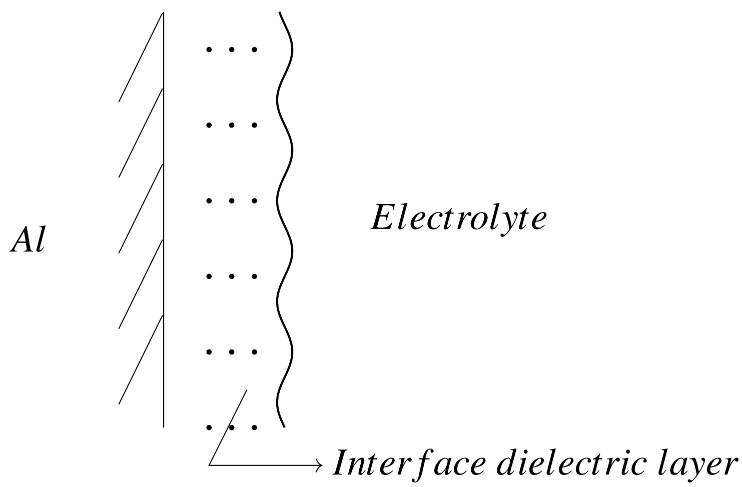


FIGURE II-A(1)

In all the circuits wherein these capacitors have been used they are represented as in Fig. II-A(2), the curved line representing the negative terminal, which here is the can containing the electrolyte.

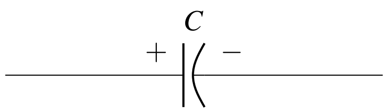


FIGURE II-A(2)

In using these electrolytic capacitors, remember to connect them with the right polarity and always below the rated voltage of the capacitor.

### 2.5.2 Appendix II-B ANALYSIS OF AN RC CIRCUIT WITH A SOURCE OF CONSTANT EMF

When a resistor and a capacitor are connected in series to a source of voltage $V_o$,

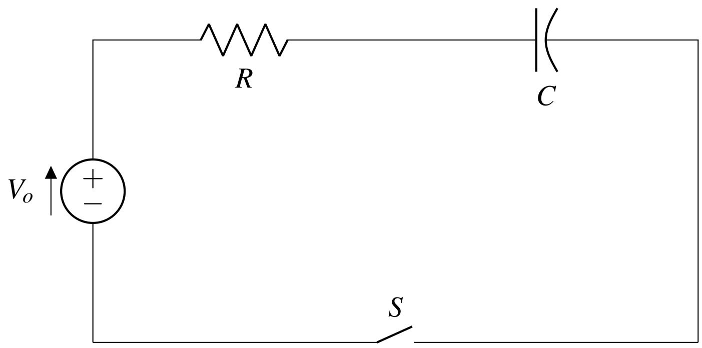


FIGURE II-B(2)

we have

$$V_R + V_C = V_0 \tag{II-B-1}$$

where $V_C$ and $V_R$ are voltages across C and R respectively . Writing

$$V_C = \frac{q}{C} \tag{II-B-2}$$

and

$$V_R = RI = R\frac{dq}{dt} \tag{II-B-3}$$

where q is the charge on the capacitor and I the current, we have

$$\frac{dq}{dt} + \frac{q}{RC} = \frac{V_0}{R}$$

This equation is readily integrated after multiplying by the integrating factor $e^{t/RC}$,

$$qe^{t/RC} = \frac{V_0}{R}\int e^{t/RC}\,dt = CV_0e^{t/RC} + A$$

where A is a constant.

For charging, we assume the initial condition $q = 0$ at $t = 0$, which establishes the equation

$$q = q_0(1 - e^{-t/RC}) \tag{II-B-4}$$

where we have put $q_0 = CV_0$

Similarly, for discharging, we set $V_0 = 0$ in Equation (II-B-1) and apply the boundary condition $q = q_0 = CV_0$ at $t = 0$ to get

$$q = q_0 e^{-t/RC} \tag{II-B-5}$$

The potential across the capacitor $(q/C)$ follows exactly, the same dependence on time as the charge .

The current is

$$I = \frac{dq}{dt} = q_0 e^{-t/RC}$$

or

$$I = I_o e^{-t/RC} \tag{B-II-6}$$

for the charging circuit and

$$I = -I_0 e^{-t/RC} \tag{II-B-7}$$

for the discharging circuit. Thus the current follows the same behaviour with time except that the sign is reversed in two cases. When the source charges the capacitor, it does work. This work is simply

$$W = \int V_0 I\, dt \tag{II-B-8}$$

since the *rate* of doing work is $V_0 I$. Using Equation (II-B-6) we have

$$W = V_0 I_o \int_0^{\infty} e^{-\left(\frac{t}{RC}\right)}\, dt = V_0 I_0 RC \tag{II-B-9}$$

Since

$$V_0 = \frac{q_0}{C} \; and \; I_0 = \frac{q_0}{RC}$$

this can be written in either of the following forms :

$$W = CV_0^2 = \frac{q_0^2}{C} \tag{II-B-10}$$

An interesting point to note is this : when the capacitor has been charged to its full potential $V_0$, it has an energy $\frac{1}{2}CV_0^2$ stored in it. Thus an energy $\frac{1}{2}CV_0^2$ has been dissipated in the resistive parts of the circuit while charging.

### 2.5.3 Appendix II-C CARBON RESISTORS

Carbon, either alone or in combination with other materials, is used in making a class of resistors which are commonly used in radio and other communication circuits. After the advent of transistors and integrated circuits where one seldom handles large power, their use has gone up phenomenally. The commonest form of mass-produced resistors is the composition resistor, in which the conducting material, graphite or some other form of carbon, is mixed with fillers that serve as diluents and combined with an organic binder. Two general types of composition resistors are the solid body, which is moulded or extruded, and the filament type, in which carbon is baked on a glass or a ceramic rod and sealed in a ceramic or bakelite tube.

Composition resistors of the usual type are, however, notoriously unstable in resistance values. If they are used only at a low power level, the change in resistance results principally from the effect of humidity on the unit. If operated near the rated load, the changes in resistance result primarily from decomposition of the organic binder.

Much better stability is found in a special film type of resistor known as a pyrolitic or "cracked-carbon" resistor. Such resistors are made by depositing crystalline carbon at a high temperature on a ceramic rod by "cracking" an appropriate hydrocarbon. In one process for making these film resistors; carbon is deposited from methane gas in a nitrogen atmosphere from which water vapour and oxygen are carefully excluded. No binder is used, and the carbon deposits consist of a hard gray crystalline form from which graphite and carbon black are completely absent. After the deposit is formed the resistor is adjusted to its required value by cutting a helical groove around the cylinder with a diamond impregnated copper wheel. This removes part of the deposit and leaves a helical conductor of suitable length and width for the desired resistance. After terminals are applied by a suitable process, the surface of a resistor is lacquered with some silicon type of varnish to provide insulation, moisture resistance and mechanical protection. These are then sorted out by measurement with a bridge in series having a tolerance of 10% or 5% or less.

For 1 watt 10 k resistors of this type, a typical temperature coefficient is -0.02% per °C (minus sign indicates a decrease of resistance with increase in temperature unlike the wire-wound resistors) and for a 5 megohm resistor this figure is -0.04% per °C.

There are some other advantages in using these film resistors. Their compactness of shape and size renders them easier to handle and suitable to fit in a small space. They are available over a wide range of resistance (from 1 ohm to 1000 megohms or more). The 10% series are available in values starting from 1 ohm in a geometric progression of about 1.5 namely, 1, 1.5, 2.2, 3.3, Similarly the values in the 5% series are in a geometric progression of 1.2 namely, 1, 1.2, 1.5, 1.8, 2.2......... The resistance values are sometimes marked with colour bands. The colour code is—
Black 0, Brown 1, Red 2, Orange 3, Yellow 4, Green 5, Blue 6, Violet 7, Gray 8, white 9.

A simple way to remember this is the *mnemonic—B.B. Roy Goes to Bombay Via Gateway. [Perhaps you could make a better one!*

You will notice four coloured bands, three narrow and one broad, on such resistors. The first two narrow ones represent the first two numbers and the third represents the number of zeros after two numbers. The three narrow bands thus give the value of the resistance. The (fourth) broad one representing the tolerance is either a silver or a gold band, the former for 10% and the latter for 5%.

Suppose on a resistor the colour bands are like this. Brown, Gray, Red and Gold. This would mean a value of 1800 i.e. 1.8 kΩ with 5% tolerance.

**REFERENCE**

Blackburn, Components handbook, Vol. 17, M.I.T. Radiation Laboratory, Mc-Graw Hill, 1949.

### 2.5.4 Appendix II-D METRONOME

A reliable source of periodic audible signals is of considerable use in the study of slowly varying phenomena. It has several advantages over the conventional method of using a stop watch. It relieves the experimenter from the difficult task of looking at the stop watch and the meter simultaneously and allows him to focus attention primarily on the latter. The data gathering also becomes relatively faster. The instrument uses a unijunction transitor 2N 2646 as a relaxation oscillator. Its basic circuit is shown in Fig. II-D (1).

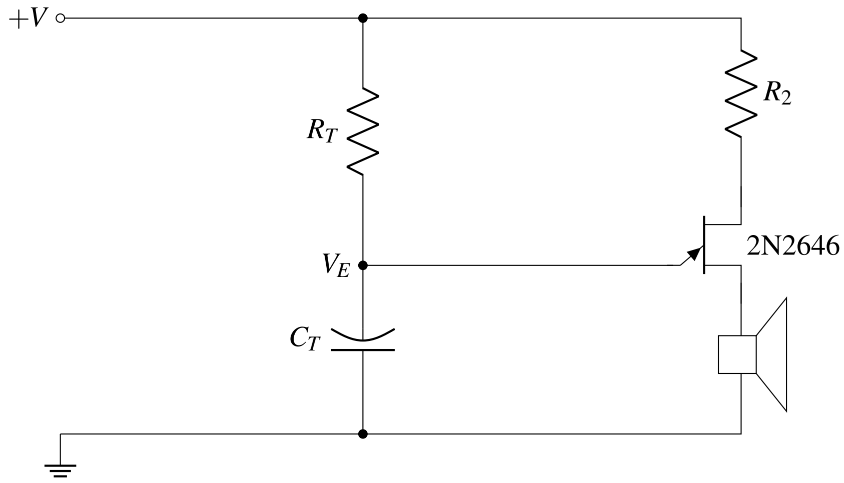


FIGURE II-D(1) Basic UJT relaxation oscillator

At the beginning of the operating cycle the emitter is reverse biased and hence nonconducting. As the capacitor $C_T$ is charged through the resistor $R_T$ the emitter voltage rises exponentially towards the supply voltage. When the emitter voltage reaches the peak point voltage $V_p$ it becomes forward biased and the dynamic resistance between the emitter and the base $B_1$ drops to a low value ($\approx 10\ to\ 15\ ohms$). The capacitor $C_T$ then discharges through the emitter and the speaker in series. When the emitter voltage reaches a low value (valley voltage $V_v$) of about 2 volts the emitter ceases to conduct and the cycle is repeated. The period is given by

$$T = R_T C_T log_e \left( \frac{1}{1 - \eta} \right)$$

where $\eta$ is the intrinsic stand-off ratio.

The intrinsic stand-off ratio varies from device to device but its variation for a particular device is only a few per cent. If the resistor $R_2$ is of the order of 100 ohms, the frequency stability of this oscillator becomes better than 1% for ambient temperatures up to $70°C$ and 10% supply variation.

The instrument uses an electrolytic $25\mu f$ capacitor for $C_T$ and the resistor $R_T$ can be varied, either in 11 discrete steps by a band switch to provide signals at intervals variable from 0.5 sec to 10.0 sec or the period can be adjusted to an intermediate value by a potentiometer. A switch selects the mode. The complete circuit is shown in Fig. 11-D (2).

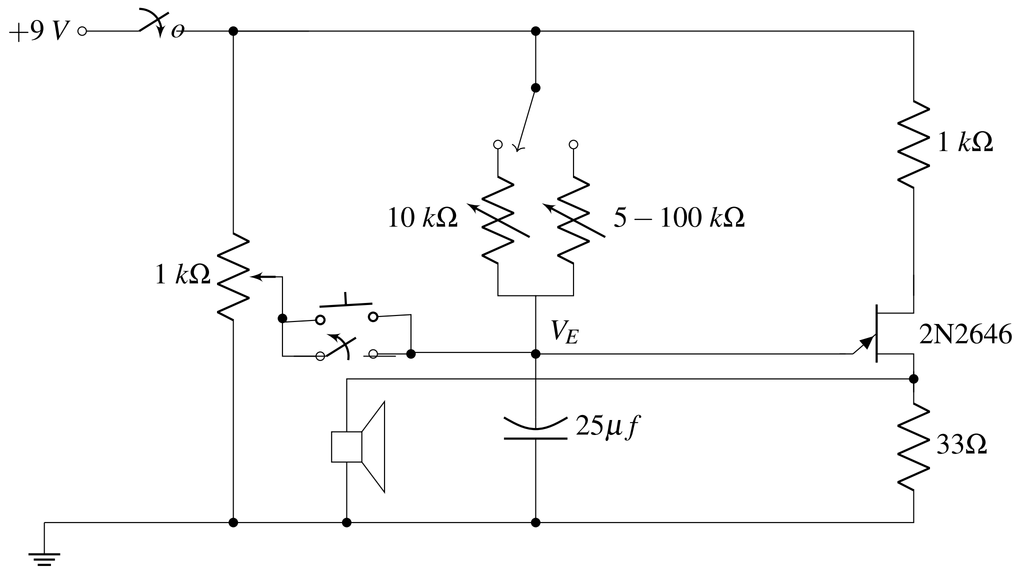


FIGURE II-D(2) Metronome with equally spaced audible ticks

The time interval between switching on and the first tick is larger than that between later successive ticks. This arises from the fact that when the circuit is switched on, the capacitor has to be charged from the ground potential to the peak point voltage $V_p$ whence it discharges giving the first tick. The capacitor retains a residual voltage of about 2 volts and it now takes a lesser time to charge to the peak point voltage $V_p$ The successive ticks have a spread of less than one percent.

The basic circuit has been modified to make the time intervals between the switching on and the first tick equal to that between successive ticks. The capacitor is given a d-c voltage equal to the valley point voltage $V_v$, before the capacitor starts the charging cycle from $V_v$ going up to $V_p$. The ticks are then synchronized to within 5%.

# CHAPTER 3

# 3. Study of RC Circuit with varying EMF

## 3.1 Introduction

We have studied the time behaviour of voltages, current etc. in RC circuits when a constant emf source is turned on or off; i.e. when the voltage input to the circuit is raised suddenly to its full value or cut off. In practical circuits, we will use the RC circuit with varying inputs. In this Chapter we would like to consider some simple cases to familiarize ourselves thoroughly with the RC circuit. In the experiments that follow, it is often necessary to charge the capacitor through a certain circuit but study its discharge in another. Such an isolation of one circuit from another is achieved quite simply with a diode.

## 3.2 Diode

Ideally, in a diode (represented as below), current will flow in the direction A to B but not in the reverse direction. Thus the forward resistance is negligible while the reverse resistance is very large (Appendix III-A).

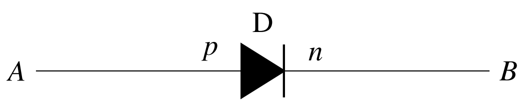


FIGURE 3.1

Thus, *the potential of the point A can be less than or equal to that of B but not more*. In practice, the forward resistance is not quite zero so that when the diode conducts the potential of A may be slightly larger than that of B. Similarly, if the reverse voltage (i.e. B positive with respect to A) is made too large, the diode will conduct.

It is better to understand the basic function of the diode in the experiments that follow with the simple circuit in Fig. (3.2) shown as the ideal case.

The source $V_0$ charges the capacitor C to its full value as soon as the circuit is complete since the diode conducts. Thus, if $V_0$ is equal to 2 volts, then, assuming R is at zero potential, P and Q are at 2 volts. If, now, the potential at P is reduced (by reducing the supply voltage $V_0$), the diode is cut off and Q will *remain at 2 volt*. On the other hand, if the potential of P is increased to a value above 2 volts, the diode will conduct and Q will go on acquiring same potential as P.

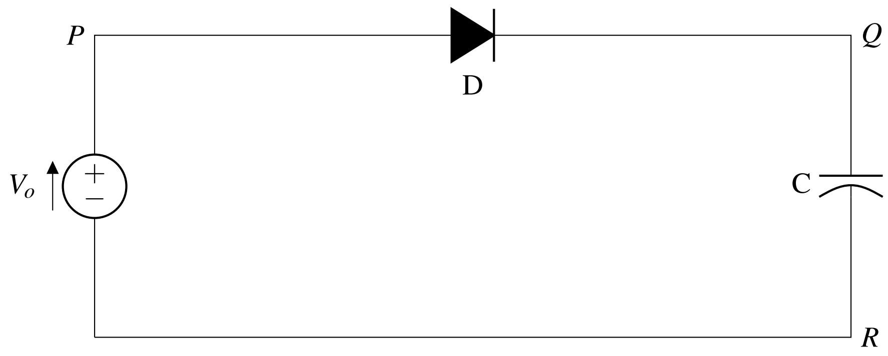


FIGURE 3.2

To be more realistic, we should always represent such a circuit with Fig. 3.3 where r represents the forward resistance of the diode plus the source resistance. This means that the charging of the capacitor cannot be instantaneous; in fact the *rise time* cannot be smaller than rC (You should familiarize yourself with the language. It means that the exponential build up of charge is determined by the time constant rC). Of course, we can make it slower by introducing more resistance in the charging circuit.

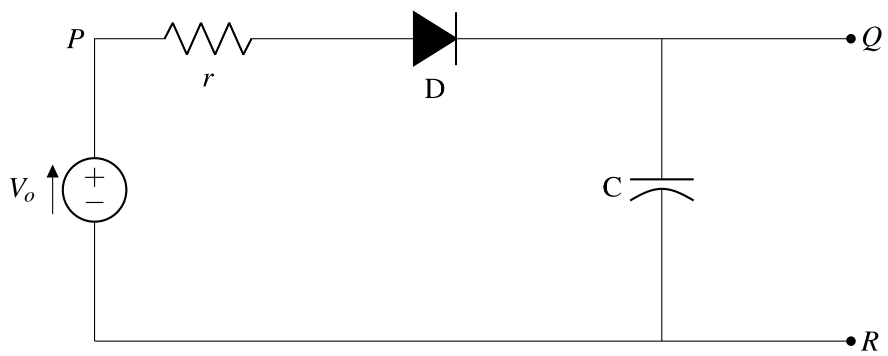


FIGURE 3.3

Should we want to measure the charge on C, we would connect a voltmeter across the points QR (Fig. 3.3). Ideally, the voltmeter should have an infinite resistance so that during a quick measurement, the charge does not decay at all. But, in practice the capacitor will discharge and the decay time will be $CR_V$, where $R_V$ is the voltmeter resistance.

These considerations should be borne in mind when performing the following experiments. If you really want a *complete* understanding of the experiments, you ought to include in your analysis, the source resistance, the forward resistance of the diode and the meter resistance; try to make at least qualitative arguments.

## 3.3 Sources of varying EMF

The most common source of varying emf that we come across in life is the a-c mains which gives an emf whose time sequence is simple harmonic in nature with a period of 1/50 sec. (Fig. 3.4).

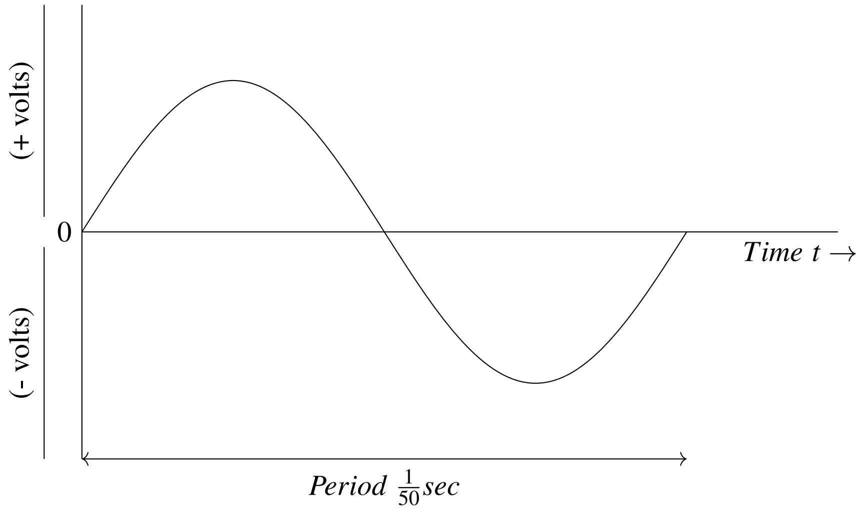


FIGURE 3.4 A Sinusoidal Pulse

In a laboratory you may also come across some oscillators of various frequencies, high as well as low. The emf generated by them may have many other wave forms such as a square wave or a saw-tooth wave (Fig. 3.5) etc.

All such sources give varying emf's. If you look at these pulses closely you would realize that the emf's are not only varying but also alternating i.e. reversing direction with time. As such these are called alternating emf's, alternating would of course, always imply variation as well.

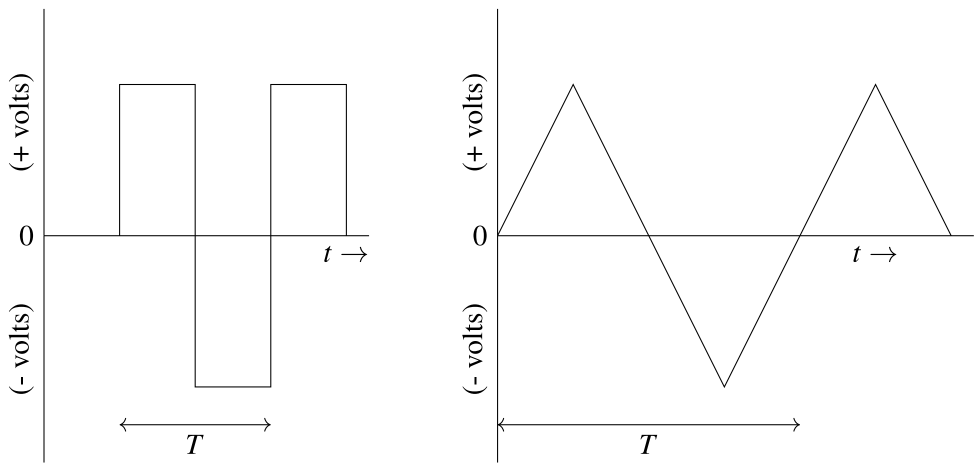


FIGURE 3.5 Alternating pulses of different shapes

To begin with, we are however, interested in unidirectional varying emf's. These can be simply produced with a d-c power-supply and a rheostat. Fig. 3.6 gives such an arrangement.

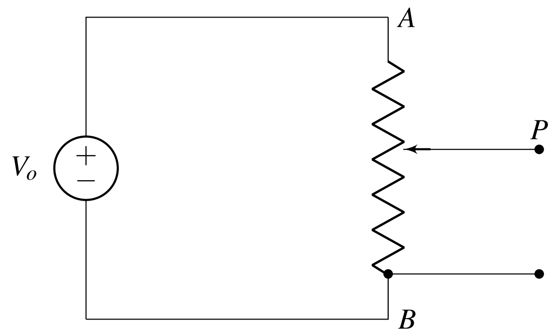


FIGURE 3.6

Connect the source $V_0$ across the rheostat AB. One of the output terminals (P) is connected to the sliding contact. As you move the sliding contact the output voltage would vary. You can thus generate any of the wave-forms like those shown in Fig. (3.7) by suitably moving the sliding contact. If you connect a voltmeter across the output terminals you should be able to verify the wave-form, provided you do not move the sliding contact rapidly. You must however, remember that owing to the inherent uncertainty involved in the manual operation of the contact you may not be able to generate a particular waveform with any amount of precision nor continue to generate it for any length of time.

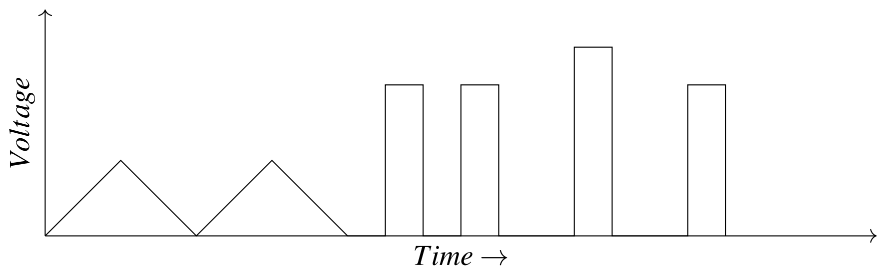


FIGURE 3.7

You can also generate an alternating emf with this device as you will see in Experiment 3-F.

## 3.4 The Network Board - 3

The board has a number of capacitors and resistors, a diode $(1N4005)$ and a d-c microammeter converted to a voltmeter of several ranges. A rheostat, 500 ohms; 0.5 amp. is also mounted on it.

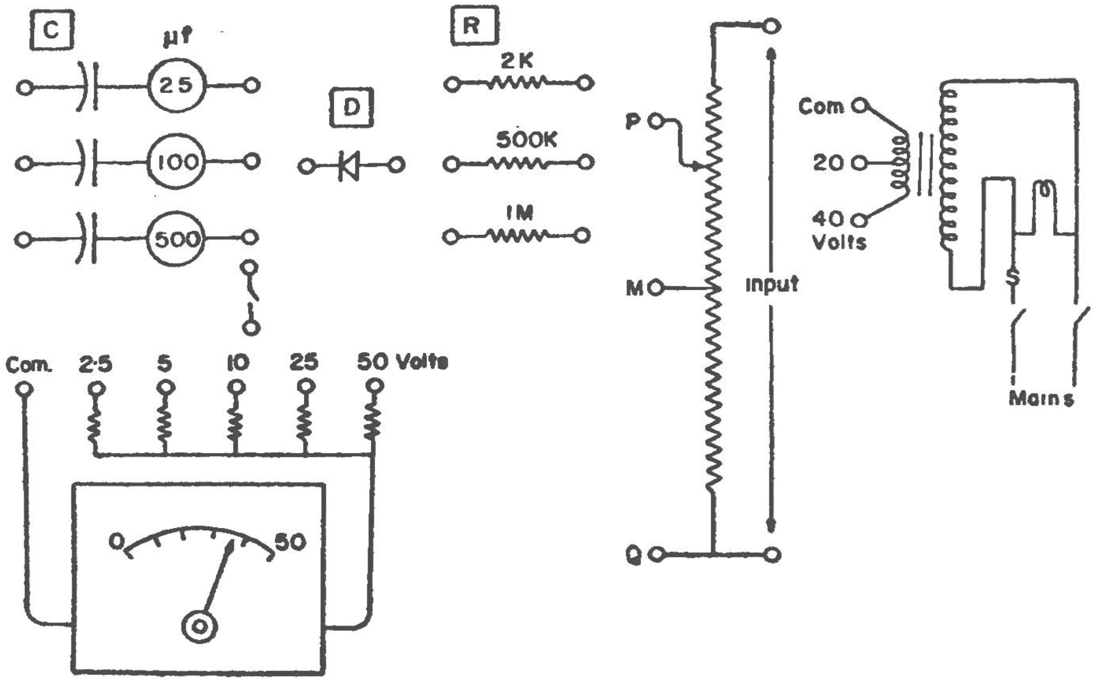


FIGURE 3.8

It is provided with a clamp to stop the slide so as to enable you to obtain any desired voltage. A small transformer giving an a-c signal of 20 volts is provided for doing experiments with alternating emf.

## 3.5 Experiments

### 3.5.1 RC Circuit with pulses of large width

#### Experiment III-A: To study the charging of a capacitor with pulses of width greater than the time constant of the circuit.

Adjust the rheostat slide so that when it is clamped at a point, it gives a certain selected voltage $V_o$ across PQ [Fig.3.9(a)].

Now connect this output to the diode and a resistor R in series with a capacitor C [Fig.3.9(b)]. The value RC should be kept small, say a few milli-seconds. (However, the charging of the capacitor will depend not only on RC but other factors as well; what other factors? For answer see Appendix III-B).

Now slide the rheostat contact from zero voltage (i.e. $V_{PQ}$) to the clamped value $V_0$ and back again to zero, the whole operation taking, say, about 2 sec. Measure the voltage developed across the capacitor.

Now discharge the capacitor (short circuit the terminals) and repeat the experiment, this time the whole operation taking considerably longer.

Now repeat the same experiment with rectangular pulses. For this slide the rheostat contact rapidly from zero to the clamped value, wait wait for a measured time( a few seconds), then slide it back to zero.

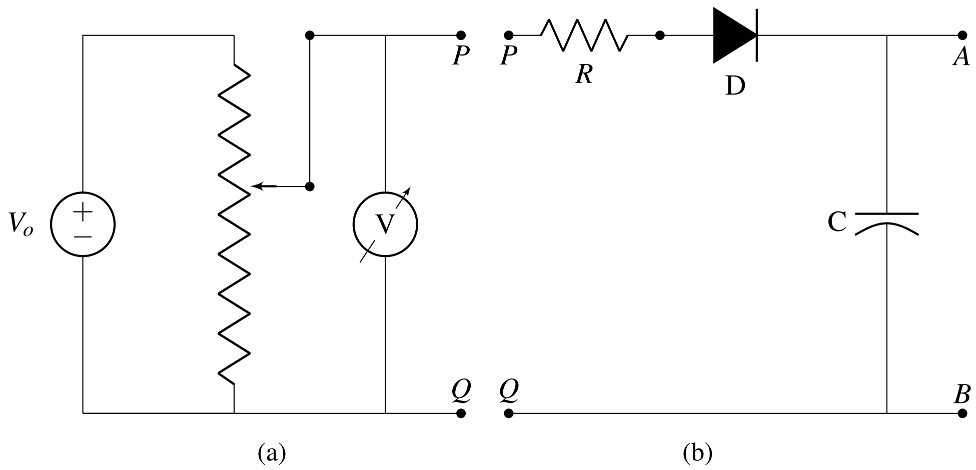


FIGURE 3.9

Try the entire operation with different capacitors.

You may notice that the charge across the capacitor as measured by the voltmeter does not quite reach the full value V. Try and understand why this is so.

### 3.5.2 RC Circuit with pulses of small width

**Experiment III-B: To study the charging with rectangular pulses of width much less than the time constant.**

Use the same circuit as Experiment 3-A but choose an RC value of about 100 sec. Now use the rheostat to give a voltage pulse of about 5 sec. width; if you do the sliding fast enough the pulse will look somewhat like Fig.3.10.

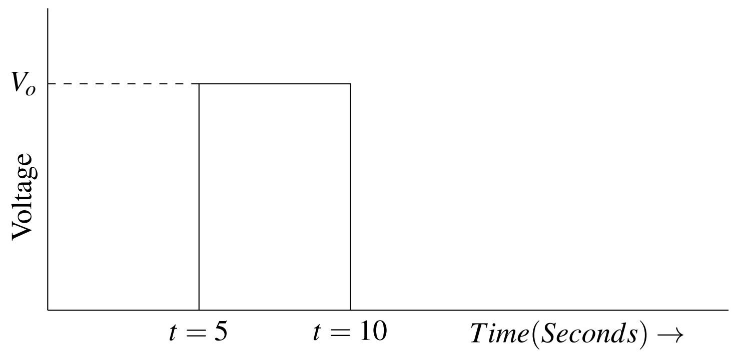


FIGURE 3.10

Since now the charging time of capacitor is very large, it will not charge to the full voltage $V_0$in this short time.This can be seen by referring to Fig. 3.11.

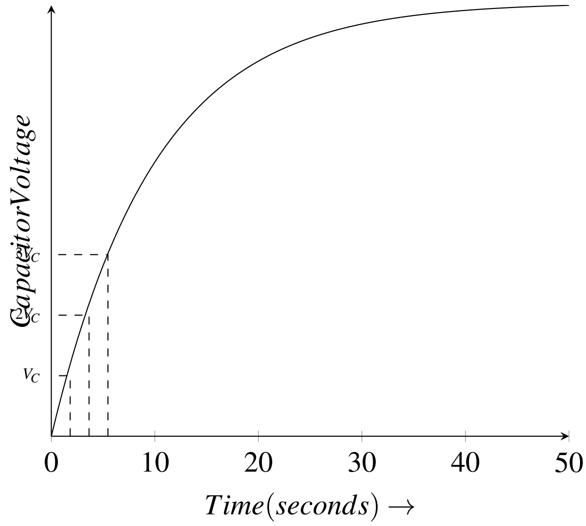


FIGURE 3.11 For $t \ll RC$, the capacitor voltage rises linearly with time

The full solid curve is the exponential rise of potential across the capacitor. But in 5 sec., the capacitor the capacitor will charge only to $V_C$ ( and diode will help it to stay there when the applied voltage falls down). Measure $V_0$ immediately after connecting the voltmeter.

Notice that, for small times, the initial portion of the charging curve is linear. (You can check this mathematically by working out

$$V(t) = V_0(1 - e^{-\frac{t}{RC}})$$

for values of $t \ll RC$)* Repeat the above measurements of $V_C$, for various pulse widths (say $5,\ 10,\ 15,\ 20,\ 25$ sec.) and plot $V_C$ as a function of the pulse width. Remember, before applying each pulse your capacitor should be completely discharged.

You will find it somewhat difficult to control the movement of the rheostat to obtain the rectangular pulse shape and width. A little practice would improve your performance.

### 3.5.3 RC Circuit with pulses of unequal widths

#### Experiment III-C: To study the charging with short rectangular of equal and unequal width ($t \ll RC$).

In this experiment, again use a large RC value (say about 100 sec.) and generate a series of 5 sec. pulses of the form shown in Fig. 3.10. It can be seen (see Fig. 3.11) that after the first pulse, the capacitor will charge to a potential $V_C$ (and stay there) and after the next pulse the potential will double and so on — as long as the total duration of all the pulses is still small compared to RC so that we stay on the linear portion.

Draw a graph between the potential and the number of pulses, determine the average value of potential rise per second of the total *rise time*. You may change the intervals between the pulses to (say) 20, 50, 200 sec., and see if the potential rise per second of pulse duration depends on these intervals.

*Putting $\frac{t}{RC} = x$ in this expression and expanding the exponential, we get

$$V = V_0\left(x - \frac{x^2}{2!} + \frac{x^3}{3!} \cdots\right)$$

. If $\frac{t}{RC}$ is very small, $x^2$ and higher powers of x can be neglected. Thus $V = V_0 x = V_0 \frac{t}{RC}$ and the potential difference across the capacitor would vary linearly with time for small values of $t/RC$. See also Appendix III-B

In fact, you may now try with a sequence of rectangular pulses whose widths are unequal, and the intervals between them are also unequal. ) keep the *pulse heights* equal). Check if the final voltage of the capacitor is proportional to the total pulse time.

### 3.5.4 RC Circuit with pulses of different shapes

#### Experiment III-D: To repeat Experiments III-A, III-B and III-C with pulses of different shapes.

In this experiment, you can try generating different shapes of pulses and repeating the experiments 3-A, 3-B and 3-C. It is unlikely that you will be able to achieve the shapes suggested with any great precision but try to study the qualitative behaviour.

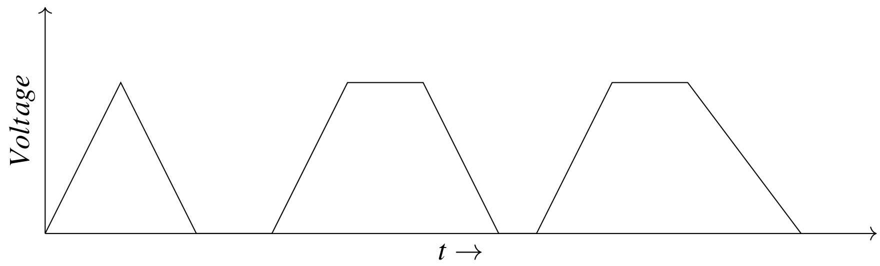


FIGURE 3.12 Unidirectional pulses of varying shapes

Some shapes you may try are shown in Fig.3.12.

### 3.5.5 RC Circuit with diode as an integrating system

#### Experiment III-E: To study an RC circuit with a diode as an integrating system.

Charge a capacitor by a number of pulses of same or various shapes and sizes for a total pulse time much shorter than the time constant of the circuit and then measure the voltage reached by the capacitor. What you read in it would be the voltage obtained on integrating all the signals.

It can be shown that the output voltage is proportional to the integral of all input voltage pulses, (see the discussion in the Appendix III-B).

$$V_C = \frac{1}{RC}\int V\,dt \tag{3.1}$$

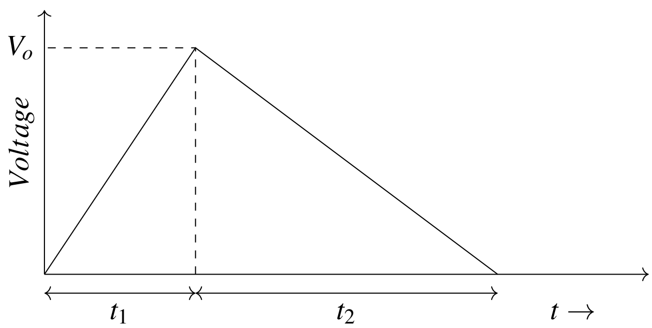


FIGURE 3.13

Thus for large RC, the voltage across the capacitor measures the time integrated value of the input voltage pulse V.

On the other hand if the RC charging time is made very small, then the capacitor quickly charges up to the peak value of the applied pulse and $V_C$ will measure this peak voltage. You should check it yourself. It amounts to charging the capacitor for a time much longer than the time constant of the circuit. Does the circuit behave as an integrating system in this case ?

For various shapes, try to determine the relationship between peak and integrated values. This will, of course, not be the same for different *pulse* shapes.

Calculate using Equation (3.1), the voltage across the capacitor when charged with triangular pulse (Fig. 3.13) of total-duration $t \ll RC$. Verify your result experimentally. (Take $V_0 \approx 20$ volts; $t = t_1 + t_2 \approx 5$ *sec*.; and $RC = 25$ *sec*).

### 3.5.6 Integrating system with an alternating input

#### Experiment III-F: To study an integrating RC circuit with alternating input.

This experiment would enable you to understand two things. One, generating of alternating pulses of various shapes and two, the rectifying action of a diode.

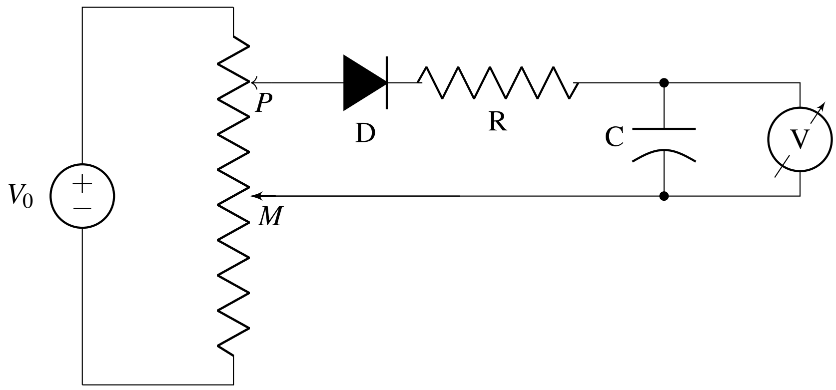


FIGURE 3.14

Instead of connecting one point of the charging circuit to one of the extremities of the rheostat wire connect it to the terminal M (Fig.3.14), its mid-point. With respect to M now the point P lying above it would have higher potential whereas all points Q lying below it would have lower potential. If you now generate rectangular pulses these would appear to be like those shown in Fig. 3.15.

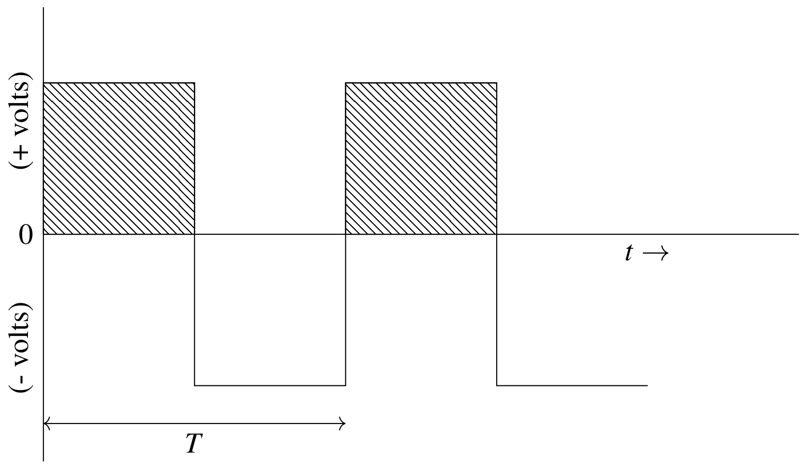


FIGURE 3.15

In this way you can generate pulses of various shapes and sizes, including pulses somewhat similar to the familiar a-c pulses (shown in Fig. 3.16).

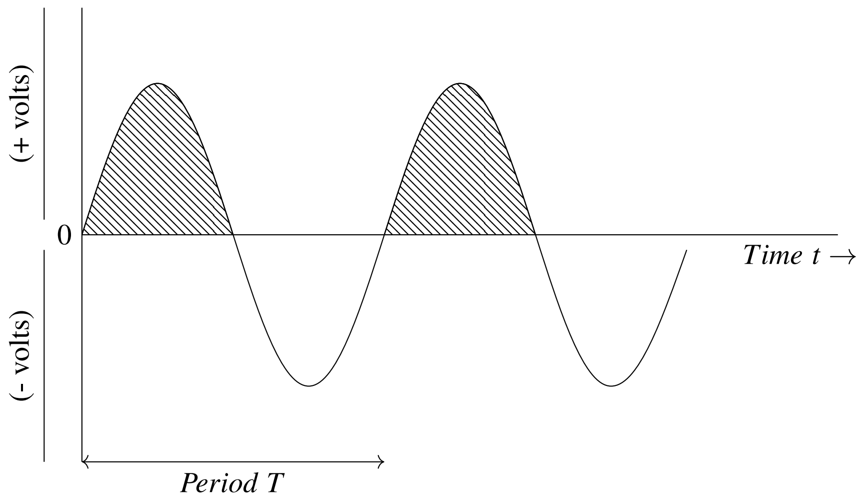


FIGURE 3.16 An alternating sinusoidal pulse

In this way you can generate pulses of various shapes and sizes, including pulses somewhat similar to the familiar a-c pulses (shown in Fig. 3.16).

In fact the sinusoidal shape is unfortunately not easy to produce with this arrangement. But in any case, you have thus produced some alternating pulses having period T.

Now you should see how the diode behaves during charging with alternating pulses. First charge by the positive half-cycle (shaded) i.e. the contact moving towards point P, staying there for a few seconds and returning to M. The capacitor gets charged to a certain voltage, which you can measure. Now Discharge the capacitor. Then charge it by the negative half of the cycle (unshaded). You will find that the capacitor does not get charged at all. If you charge the capacitor by a number of cycles (say 5), you will find that the voltage acquired by the capacitor would be only that due to 5 positive half-cycles. Try to verify this and understand the action of the diode.

What voltage(charge) would you expect on the capacitor if it were to be charged with both the positive and negative halves of a pulse? You can work it out easily in the case of rectangular pulses (Fig.3.15) using Equation (3.1). If you try that, you will get an interesting result. You can now understand what happens to a capacitor when charged in a d-c circuit and in an a-c circuit with and without a diode.

### 3.5.7 Integrating system with sinusoidal input

#### Experiment III-G: To study an integrating system with a sinusoidal input.

Having understood the meaning of alternating pulses, their production and the rectifying action of the diode, the experiment can now be repeated with sinusoidal alternating pulses. Instead of the d-c source and the rheostat connect the two points AB of the charging circuit directly across the transformer provided on the board (Fig. 3.17).

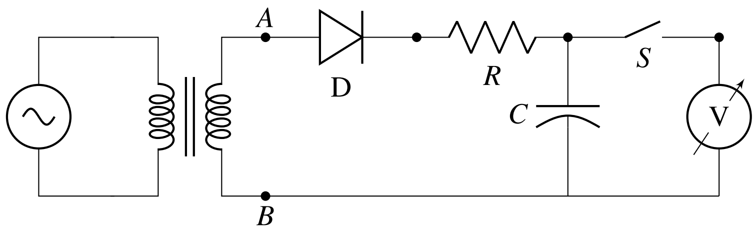


FIGURE 3.17

The transformer gives sinusoidal pulses of frequency 50 hertz at about 20 volts. *You must not draw a current of more than 100 mA under any circumstances from the transformer.* You will have to remember that owing to high frequency of these pulses (50 per sec.) or small period ($1/50$ sec.) it would not be possible to isolate individual pulses for measurement. Having understood the action of the diode you can charge the capacitor for different times (5, 10, 15, 20...... sec.) and plot $V_C$, against t or against the number of positive half-cycles.

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## 3.6 APPENDIX

### 3.6.1 Appendix III-A DIODE

The diode we use in all these boards is a particular semiconductor device known as a junction diode. We shall not describe the physics of semiconductors and refer the reader to any number of standard texts on the subject.

What is of practical interest to us is the so called V-I characteristic of the diode. It suffices for us to know that it consists of a so called p type material joined to an n type material (See Fig. 3.1). If a voltage is applied on the diode making the p end positive with respect to the n end, it is said to be forward biased and if p is made negative relative to n, it is reverse biased. The resulting current through the diode is typically as shown below [Fig. III-A(I)].

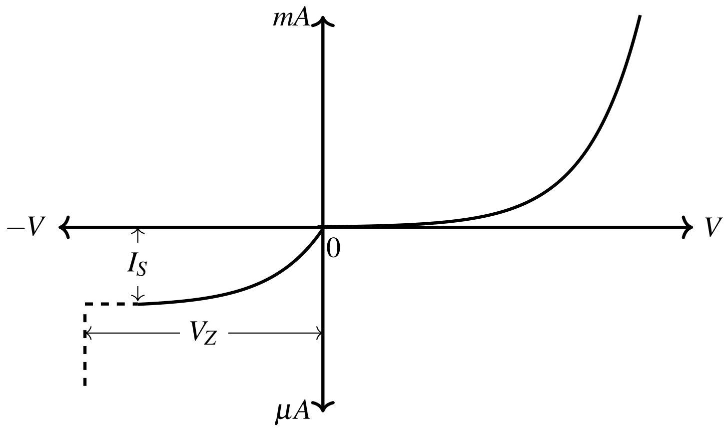



FIGURE III-A(1) General form of forward and reverse characteristics of diode

;

a) For positive voltage the current is in milliamperes, whereas for negative voltages it is in microamperes. This means that for *reverse bias* the diode resistance is very much higher than for forward bias.
b) For forward bias $\frac{V}{i}$ ( the resistance) falls with increasing V. At about 1 Volt(not shown) the resistance drops to about 15 ohm in a typical diode.
c) For reverse bias $\frac{v}{i}$ increases with increasing magnitude of V. Typical resistance becomes a few megohm.
d) For large reverse bias (shown as $V_Z$) a large current starts flowing. $V_Z$ is known as the breakdown voltage. For the 1N4005 diode used in our network boards its value is 750 volt, but it is much smaller for several diodes.

To summarize, the forward resistance as long as the voltage exceeds, say 1 volt, is of the order of 10 ohms or less. The back resistance is several megohms.

There is, however, a cautionary remark we make. Should the reverse bias exceed about 500 volts (these figures vary with the type of diode), then a large current does flow and the diode is said to break down; often this will damage it permanently. One should avoid exceeding this reverse voltage.

In using these diodes, it is desirable to keep the forward current to be in the region of tens of milliamperes or more and the other resistances in the circuit much larger than (say) 100 ohms. We can then safely neglect the resistance of the diode and the voltage drop across it. Large forward current may damage the diode. For the 1N4005 diode the safe forward current is 1 amp.

### 3.6.2 Appendix III-B DERIVATION OF EQUATION ON VOLTAGE INTEGRATION

Let us first examine the circuit of Fig.3.9(b). If the capacitor voltage is $V_C$ and signal voltage V, the essential point about the diode is that its resistance r is negligible (compared with R) for $V > V_C$ and practically infinite for $V < V_C$. An important result is that when V falls below $V_C$ the capacitor retains the potential $V_C$, like memory.

In Appendix II-B the differential equation for the CR circuit was written as

$$\frac{dq}{dt} + \frac{q}{RC} = \frac{V_0}{R}$$

Here we shall modify it in two respects. First, we replace $V_0$ by V, which need not have a constant value. Second, we recall that the equation applies now only when $V > V_C$, because of the presence of the diode. Hence we write

$$\frac{dq}{dt} + \frac{q}{RC} = \frac{V}{R} \qquad (for\, V > V_0) \tag{III-B-1}$$

Using the integrating factor $e^{t/RC}$ and integrating, we get (as in Appendix II-B)

$$qe^{t/RC} = \frac{1}{R}\int Ve^{t/RC}dt$$

For the capacitor voltage $V_C = q/C$ this becomes

$$V_C e^{t/RC} = \frac{1}{RC}\int V_c e^{t/RC}dt \tag{II-B-2}$$

The integration will cover those times only during which $V > V_C$. For other times the integral will contribute zero, because $R + r = \infty$.

Consider a rectangular pulse of height $V_1$ applied for time $t_1$, the initial voltage of capacitor being $V_C' < (V_1)$. For this case the solution of Equation (III-B-2) is

$$V_C e^{t_1/RC} = V_1 e^{t_1/RC} + (V_C' - V_1) \tag{III-B-3}$$

The last term $(V_C' - V_1)$ comes from the boundary condition $V_C = V_C'$ at t=0. This result can be rearranged as

$$V_C = V_C' e^{-t_1/RC} + V_1(1 - e^{-t_1}/RC) \tag{III-B-4}$$

This solution means that for $t_1 \gg> RC$ the first term becomes zero and the second term becomes $V_1$. Thus for $t_1 \gg RC$ the capacitor 'forgets' the initial voltage $V_C'$ with which it started and reaches the magnitude $V_1$ of the applied pulse, irrespective of $V_C'$ as well as $t_1$.

However, for $t_1 \ll RC$ Equation (III-B-4) can be rewritten in the more useful form

$$V_C = V_C' + \left(V_1 - V_C'\right)\frac{t_1}{RC} \tag{III-B-5}$$

It says that the memory $V_C'$ is retained, and to it is added a quantity proportional to $(V_1 - V_C)t_1$.

If we now add the condition $V_1 \gg V_C'$,apart from $t_1 \ll RC$, Equation (III-B-5) takes the form

$$V_C = V_C' + V_1'C \tag{III-B-6}$$

Thus a voltage equal to $(1/RC)$ times the voltage-time product of the pulse is added to the initial voltage $V_C'$.

If we now start from initial potential zero and apply several rectangular pulses of heights $V_1$, $V_2$, $V_3$.... and widths $t_1$, $t_2$, $t_3$...respectively, we can now see that

$$V_C = \frac{V_1 t_1 + V_2 t_2 + V_3 t_3 + \ldots}{RC} \tag{III-B-7}$$

provided that (i) each of $t_1$, $t_2$, $t_3$ ... is much smaller that the charging time constant RC of the circuit, and (ii) each of $V_1$, $V_2$, $V_3$ ... is much larger that the voltage already attained by the capacitor at the time of application of that pulse.

Now a pulse of any shape can always be considered as the sum of several rectangular pulses of varying heights. Therefore Equation (III-B-7) can be generalised to

$$V_C = \frac{1}{RC}\int V\,dt \qquad \text{(III-B-8)}$$

In this sense the circuit of Fig.3.9(b) is an integrating circuit. It integrates voltage over time, provided that

i). RC of the circuit is much larger than the time over which voltage integration is desired.

ii). The capacitor voltage does not exceed a small fraction of the heights of the voltage pulses involved.

Actually the two conditions are inter-linked. Large RC not only makes the approximation $1 - e^{-t_1/RC} \approx t_1/RC$ more valid, it also keeps $V_C'$ low so as to increase the validity of the approximation $V \gg V_C'$.

IV

# CHAPTER 4

# 4. Study of RC Circuit with a Low Frequency a-c Source

## 4.1 Introduction

We now proceed to study the behaviour of an RC circuit in detail when the impressed voltage is sinusoidal. The most convenient source is the line voltage which, in our country, is supplied at frequency 50 hz, which means period 0.02 sec. Unfortunately, this period is too small for looking at simple a-c phenomena, directly to watch the current swing back and forth, to see the phase relationship between different voltages in a circuit and so on. In fact, if you try to read an a-c voltage in a d-c meter you will see that it gives no deflection, the signal swinging back and forth too fast for the needle to follow. One way to avoid this difficulty is to use a cathode ray oscilloscope. The a-c signal can be displayed and one can make all the necessary measurements. You should, in due course, learn such techniques.

The experiments described in this chapter have been designed so that you can study the properties of an a-c circuit using d-c meters. The trick is to use extremely low frequencies. The circuit you will study is a simple RC circuit and the source is an ultra low frequency simple harmonic oscillator (Appendix IV-A) generating alternating voltages in the frequency range 0.1 to 10 hz. At such frequencies the d-c meters will easily 'follow' the signal.

## 4.2 Phase

You have learnt how we can arrange to measure, in principle, peak values of pulses of various shapes. We shall now concentrate on the sinusoidal pulses alone. An a-c voltage signal may be represented by the relation

$$V(t) = V_0 \; sin \; (\omega t + \phi) \tag{4.1}$$

where $V_0$ is the peak value, $\omega = 2\pi$f, where f is the frequency*, and $\phi$ is the *phase* of the input at t = 0.

What do you understand by the term 'phase'? Look at the following alternating signal. Pay particular attention to the voltage at time t=0.

Obviously, this voltage value is not 0 at t=0. Equation (4.1) yields, for t=0,

$$V(0) = V_0 \; sin \; \phi$$

*$\omega$ is called *'angular frequency'*, as distinct from the frequency f.

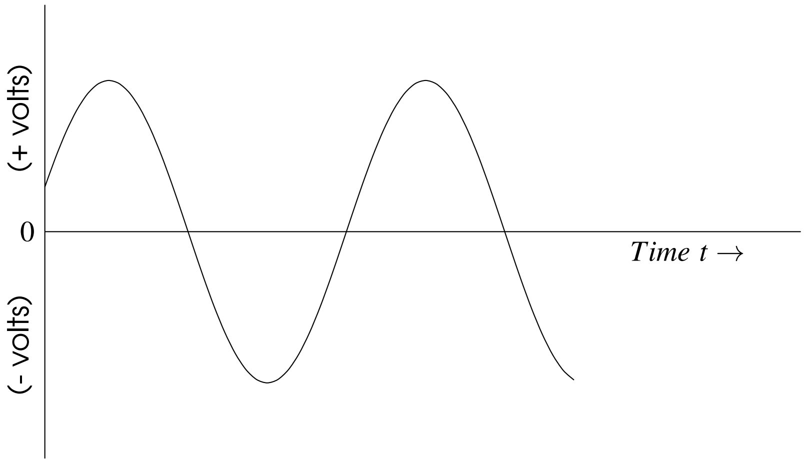



FIGURE 4.1 Alternating signal with phase not zero at t=0

Thus $\phi$, along with $V_0$, gives you the value of V(t) at t=0. Now look at the following signal (Fig. 4.2).

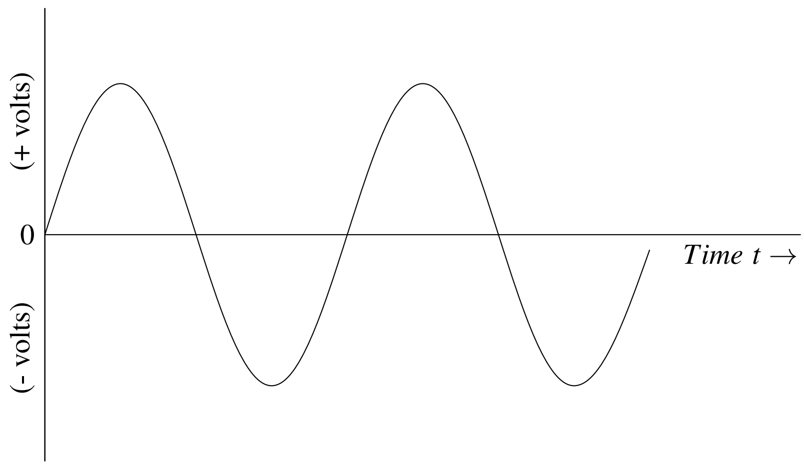



FIGURE 4.2 Alternating sinusoidal signal with phase zero at t=0

Here the voltage value is zero at t=0 and hence according to Equation (4.1) $\phi$ must also be zero. The value of $\phi$ thus depends on the choice of the zero of time. If we are concerned only with one such voltage, we can always choose the zero of time such that $\phi$ = 0 (i.e. V = 0 at t = 0). If there are two such signals $V_1(t)$ and $V_2(t)$, we cannot in general choose the zero of time such that both $V_1$ and $V_2$ are zero at t=0. We, therefore, stick to the general form of Equation (4.1).

Thus even if the peak values and frequencies of $V_1$ and $V_2$ are the same, their phases $\phi_1$ and $\phi_2$ may be different[†] In fact let us think of a voltage $V = V_0\ sin\ (\omega t + \phi_1)$ and a current $I = I_0\ sin\ (\omega t + \phi_2)$ as shown in Fig. 4.3. Here V and I are different quantities; yet the phase difference between them $\phi_1$ -$\phi_2$ has great significance. In the figure we measure the time difference $\Delta t$ at which corresponding states are reached by V and I. Here typical corresponding states are a and $a'$ (both peaks), b and $b'$ (both bottoms), c and $c'$ (both zeros, slope negative), d and $d'$ (both zeros, slope positive), e and $e'$ (both half of respective peak values)—and all these pairs give the same $\Delta$ t.

[†]Strictly speaking, *phase* refers to the entire argument $\omega t + \phi$. We shall often use the term 'loosely' to mean the phase at t=0. In fact our interest often will be in the *phase difference* between two alternating signals; since we deal with quantities of same $\omega$, the term $\omega t$ will cancel out in the difference.

The phase difference $\Delta\ \phi$ (i.e. $\phi_1$— $\phi_2$) is given by

$$\Delta\phi = \omega\Delta t$$

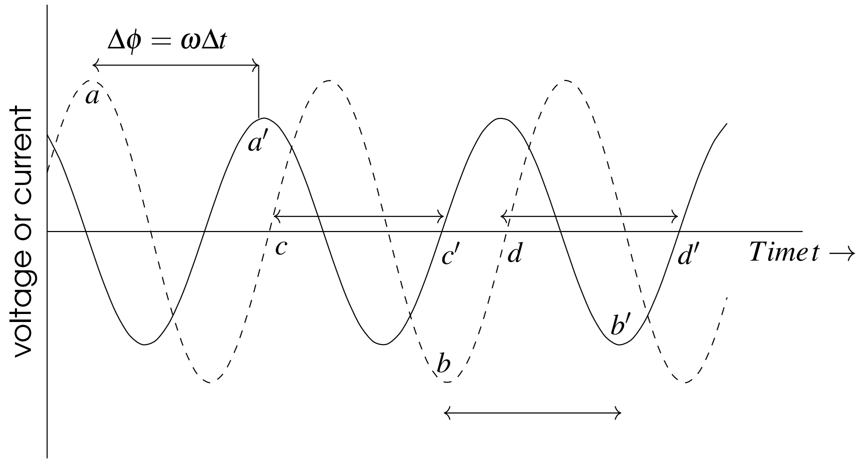



FIGURE 4.3

When the input in a circuit is of a particular frequency, all currents and voltages in the circuit will be of the same frequency since the circuit is a linear one. There can be non-linear circuit elements which can change frequencies but we shall not consider these.

There, then, are two parameters for each physical variable in a.c. -the peak value (the amplitude) and the phase or, to be more precise, the phase difference between any pair of a-c components in the circuit. It is important to appreciate this basic fact and to distinguish this from the situation in d-c. It is very much the aim of this series of experiments to develop a familiarity with the concept of phase difference between various quantities and an appreciation of its importance.

Although we have used the sine form to describe an a-c signal, we could just as well have used the cosine form. It will turn out that the relative phases of voltages and currents in an a-c circuit are identical whether we use the sine or the cosine form. The cosine form, however, has an advantage. If we assume the source voltage to be of the form $V_0\ cos\ \omega t$ the transition to d-c is readily made by letting $\omega \to 0$. For this reason, we shall use the cosine form from now on.

## 4.3 Vector Diagrams

Suppose we consider the superposition of two signals $V_1(t) = V_{10}\ cos\ (\omega t + \phi_1)$ and $V_2(t) = V_{20}\ cos\ (\omega t + \phi_2)$ of the same frequency. Then for the resultant signal one writes

$$V(t) = V_1(t) + V_2(t) \tag{4.2}$$

Substitution and a little algebra will show that

$$V(t) = V_0\ cos\ (\omega t + \phi) \tag{4.3}$$

where

$$V_0^2 = V_{10}^2 + V_{20}^2 + 2V_{10}V_{20}\ cos\ (\phi_1 - \phi_2) \tag{4.4a}$$

and

$$tan\ \phi = \frac{V_{10}\ sin\ \phi_1 + V_{20}\ sin\ \phi_2}{V_{10}\ cos\ \phi_1 + V_{20}\ cos\ \phi_2} \tag{4.4b}$$

Thus, the superposition gives again an a-c signal of the same frequency but with a different amplitude and phase.

The values of $V_0$ and $\phi$ can be arrived at elegantly with the help of a graphical method which represents these voltages by vectors. In Fig. 4.4(a) the line OA represents the potential $V_1$ the length of OA giving its peak value $V_{10}$ and the angle which OA makes with a fixed line OX gives the phase $\phi_1$ (at $t = \theta$).

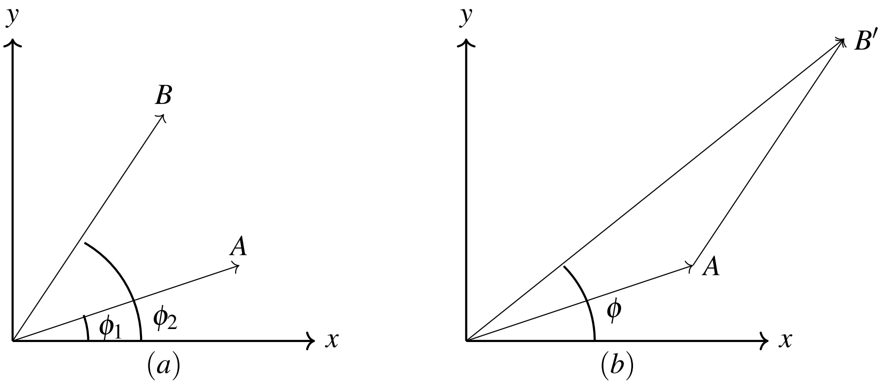



FIGURE 4.4 Superposition of two signals by graphical method

Similarly OB represents $V_2$ in magnitude and phase. The result of super- position of $V_1$ and $V_2$ is then represented by the vector sum of OA and OB which may more easily be obtained by the triangle in Fig.4.4(b).

Here, AB' is drawn parallel and equal in length to OB. The resultant is OB' and the phase is just the angle which OB' makes with OX.

Actually, the diagrams 4.4 (a) and (b) represent the situation at t = 0. Suppose we now imagine all the vectors [such as the triangle OAB' in Fig. 4.4(b) rotating in the xy plane around fixed point O with constant angular velocity $\omega$. Then the angles that OA, AB' and OB' make with OX at a general time t will be

$$\phi_1 + \omega t, \phi_2 + \omega t \; and \; \phi + \omega t$$

respectively so that the projections of OA, AB' and OB' on OX will give $V_1(t)$, $V_2(t)$ and $V_3(t)$.[‡]

## 4.4 Analysis of an RC circuit

If a resistor R and a capacitor C are in series with an a-c source, we can represent the potential difference across R and C by a vector diagram. It can be shown (Appendix IV-B) that the voltage across the capacitor is 90 behind in phase compared to the voltage across the resistor. OR represents $V_R$, OC represents $V_C$ and the vector sum represents the applied voltage $V_0$.

[‡]It may be noted that $V_1(t)$, $V_2(t)$ and $V_(t)$ are *ot vectors*. Similarly the currents, added by the same process are not vectors. The method of Fig. 4.4, only shows that the addition of two sinusoidally varying quantities of the same frequency can be achieved by drawing vector lines in two dimensions to represent them with proper *phases*. For this reason such diagrams are also called *phasor diagrams*

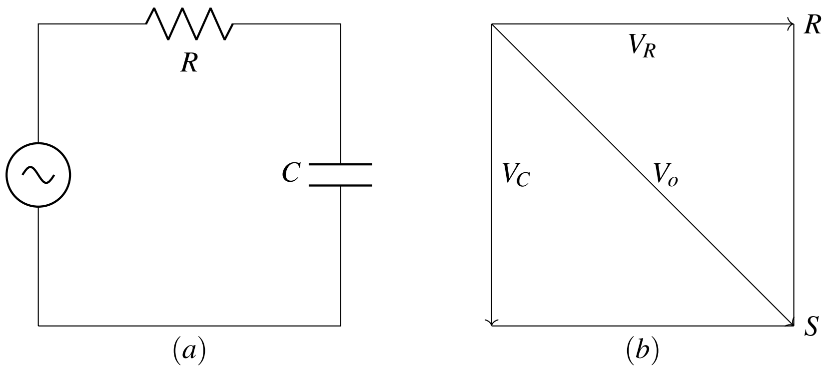



FIGURE 4.5

In Fig. 4.5(b) the potential $V_C$ across the capacitor has been shown 90 clockwise [§] relative to the potential $V_R$ across the resistor. One frequently uses the expression ‘the voltage across the capacitor lags behind the current’. This is because the phase of current is same as the phase of $V_R$. In using such language, one does not imply that voltages and currents can be represented as vectors in the same diagram—obviously, they are different classes of quantities. Nevertheless, the idea of a voltage lagging behind or leading a current is widely used and should not create any confusion.

The relevant mathematics is relegated to the Appendix where it belongs. Mathematics is important and you should try to understand it. But it is much more important to learn how to use the results.

## 4.5 The Network Board - 4

There are three centrally pivoted d-c meters (25-0-25 $\mu A$) on the board. Each one of them can be used either as a multiple range current meter or a voltmeter. The ranges are marked on plates below the terminals. A bridge of diodes and a capacitor are also provided in case you wish to use these for measurements with higher frequencies (say 10 hz or more). With lower frequencies, of course, the d-c meters can be used just as they are. An important feature of these meters is that they are *critically damped*[¶]. A mirror is provided for observing readings on any two adjacent meters simultaneously, so that you can measure the phase relationship between the two signals. All you are required to do is to clamp the mirror over one meter and adjust the tilt of the mirror until you can see the reflected image of the needle of the other meter along with the needle of the first. With a little practice you can read the two meters almost simultaneously, within about 0.1 sec. Also provided with each meter is a diode which can be used to make the meters move in one direction.

[§] In vector ( or phasor) diagrams ’clockwise’ means lags behind in phase. Thus in Fig. 4.5(b) $V_C$ is clockwise relative to $V_R$, means $V_C$ lags in phase; $V_R$ is anticlockwise relative to $V_C$ means $V_R$ leads in phase (relative to $V_C$)

[¶] An under-damped system reaches equilibrium after many oscillations. An over-damped one does not oscillate, but moves *slowly* to equilibrium position. A critically damped system moves fastest to the final reading without oscillations.

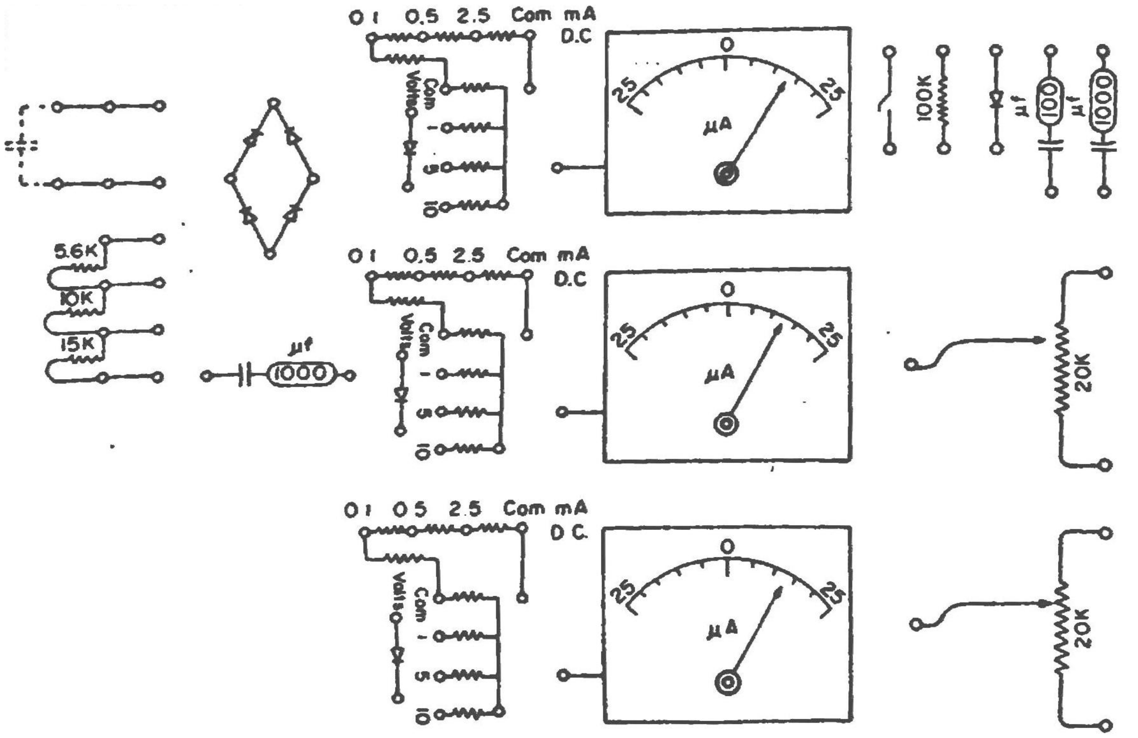



FIGURE 4.6

Fig. 4.6 gives the schematic layout of this board while Plate II shows an actual photograph. You can see two capacitors, a diode, a resistor and a switch in the top right corner. These are meant for use when you wish to measure the peak values of different voltages in the circuit. At the lower right of the board you will find two potentiometers, each of resistance $20k\Omega$, with graduated scales.These are used in measuring the peak or instantaneous values of voltages. A few oil-filled capacitors and an ultra- low frequency oscillator are among other things supplied with the board.

## 4.6 Measurement of Peak and Average a-c Voltages

The principle would be clear from the circuit in Fig. 4.7. Using the lower value of capacitance ($100\mu$f) gives time constant RC = 10 sec. If the capacitor is allowed to charge ($S_1$ on, $S_2$ off) for about 50 sec. or more, it reaches the peak input voltage $V_0$. If you now discharge it through the meter ($S_1$ off, $S_2$ on) you have a measure of the peak voltage .

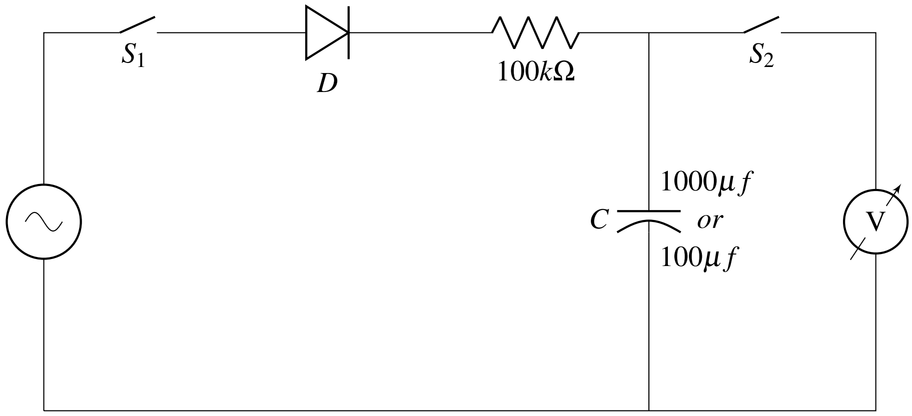



FIGURE 4.7

You can also measure the peak values by a 'null method'. The idea is this. Suppose you set the potentiometer connected across the 9 volt battery (Fig. 4.8) to give a d-c voltage of x volt. Looking at Fig. 4.8, it is clear that current will flow through the meter only if the point A is at a potential exceeding x volt.

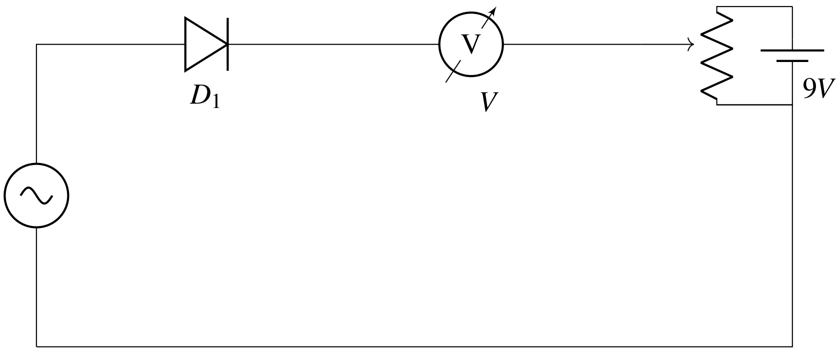



FIGURE 4.8

Thus, if the a-c peak voltage exceeds x volt, you will get a 'kick' in the meter for a part of the time. If the a-c peak voltage is below x volt you will never get a kick in the meter. Figure 4.9 shows two d-c voltages.For x=d-$C_i$, the meter registers a 'kick' between times $t_1$ and $t_2$, $t_3$ and $t_4$ and so on.

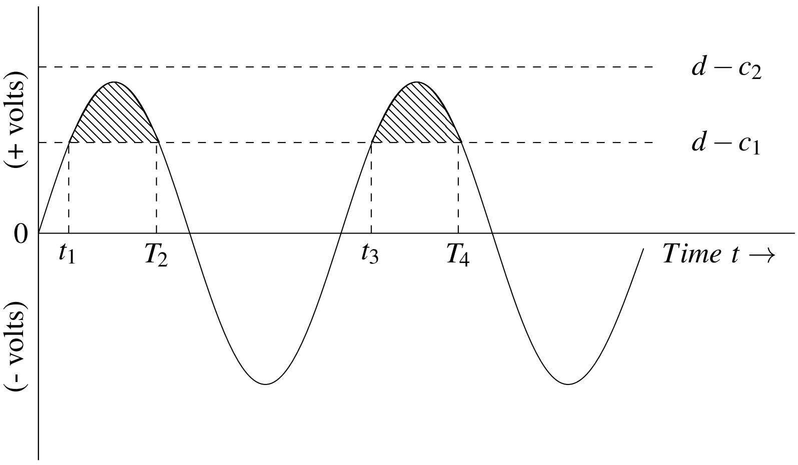



FIGURE 4.9

If, however, the d-c voltage is set higher than the a-c peak (the d-$c_2$ signal), no such 'kick' should be observed. Trying a few d-c values, one can easily measure the peak a-c voltage. It equals that d-c voltage at which the meter just ceases to show kicks.

The measurement of instantaneous voltage will be discussed in Experiment 4-B.

## 4.7 Experiments

### 4.7.1 RC circuit - Voltages and phases

#### Experiment IV-A: To make a preliminary study of voltages and phase relationship in a simple RC circuit.

We start by studying the response of a simple RC circuit. Set the oscillator to give you the lowest possible frequency (about 0.1 hz) and make up the connections as shown in Fig. 4.10.

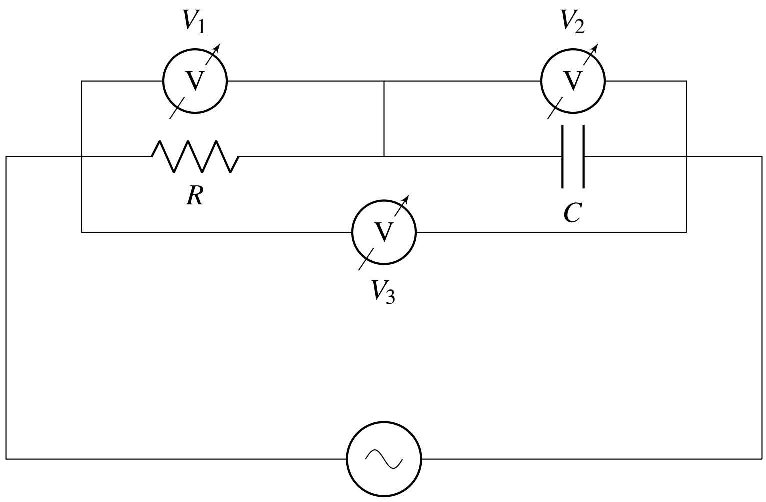



FIGURE 4.10

i. Watch the three meters and give a qualitative description of their deflection with time[‖]. Note, particularly, the relative phases of the three voltages, i.e. when they pass through maxima relative to each other.
ii. Measure the three voltages by the two methods outlined in the introduction, the null method of comparison with a d-c voltage , the 'peak' method.
iii. Now change the voltage amplitude of the oscillator output and see if the ratios of the three voltages change proportionately. Do the relative phases change or remain the same?

### 4.7.2 A purely resistive, a purely capacitative and mixed circuits

#### Experiment IV-B: To study the voltages in a purely resistive, a purely capacitative and a mixed circuit.

The purpose of this experiment is to study clearly the distinction between circuits with purely resistive elements, purely capacitive elements and circuits with both.

i. Connect three resistors in series and measure the peak voltage across each ($V_{R_1}$, $V_{R_2}$ and $V_{R_3}$ ) Verify the relation

$$V_{R_1} + V_{R_2} + V_{R_3} = V_{R_1R_2R_3} \tag{4.5}$$

ii. Similarly, for a circuit of three capacitors in series , verify that

$$V_{C_1} + V_{C_2} + V_{C_3} = V_{C_1C_2C_3} \tag{4.6}$$

[‖]You will find that you have to wait for a while (about a minute) for the *transients* to die out. Your main interest in this experiment is in steady state; but for a complete understanding of the behaviour, you ought to discuss the transient response as well. A brief discussion is given in Appendix IV-C. Can you see if the voltage $V_2$ leads or lags by how much? Try and depict $V_1$, $V_2$ and $V_3$ on a vector diagram

iii. Now make up a circuit of two resistors and two capacitors in series (Fig. 4.11).

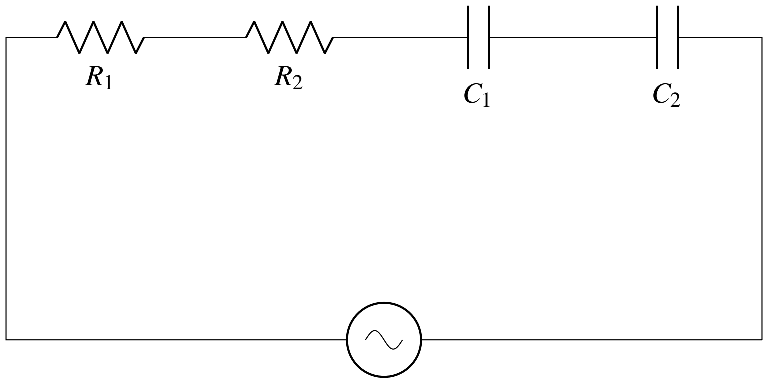



FIGURE 4.11

You should satisfy yourself that

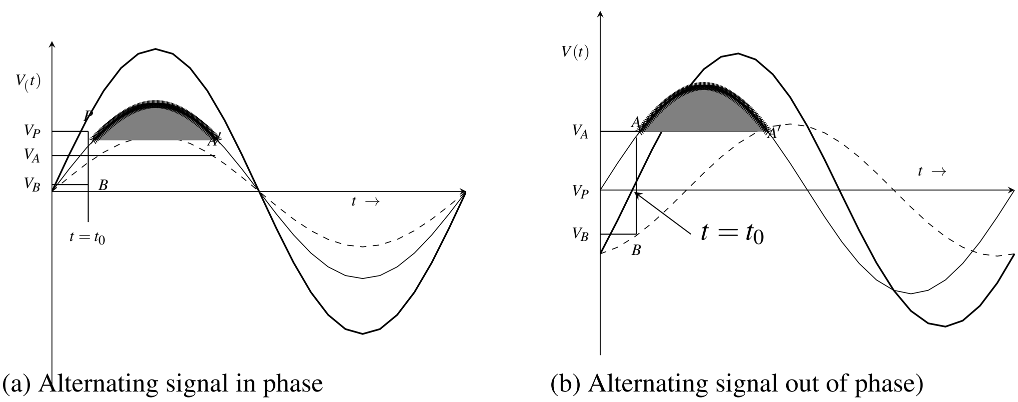
$$V_{R_1} + V_{R_2} = V_{R_1R_2} \;\; and \;\; V_{C_1} + V_{C_2} = V_{C_1C_2}$$

but

$$V_{R_1R_2} + V_{C_1C_2} \neq V_{R_1R_2C_1C_2}$$

All, of course, would be entirely obvious if you construct the relevant vector diagram of the voltages in each case.

Calling $V_{R_1R_2} = V_R$ and $V_{C_1C_2} = V_C$ and $V_{R_1R_2C_1C_2} = V_o$, you can construct the voltage triangle from these *measured* peak values and verify that $V_R$ and $V_C$ have a phase difference of $90^o$.




(a) Alternating signal in phase

(b) Alternating signal out of phase)

FIGURE 4.12

The result that the algebraic sum of $V_R$ and $V_C$ does not turn out to be equal to the emf $V_0$ in the circuit appears to be in violation of the Kirchhoff's law. Before we comment further on this let us repeat these observations measuring this time the instantaneous values of the voltages. Let us first define our problem with the help of an illustration. Fig. 4.12 shows two alternating signals in and out of phase. Suppose you want to measure the voltage values at time t = $t_0$. In other words, you want to know $V_A$ and $V_B$. You can do this as follows:

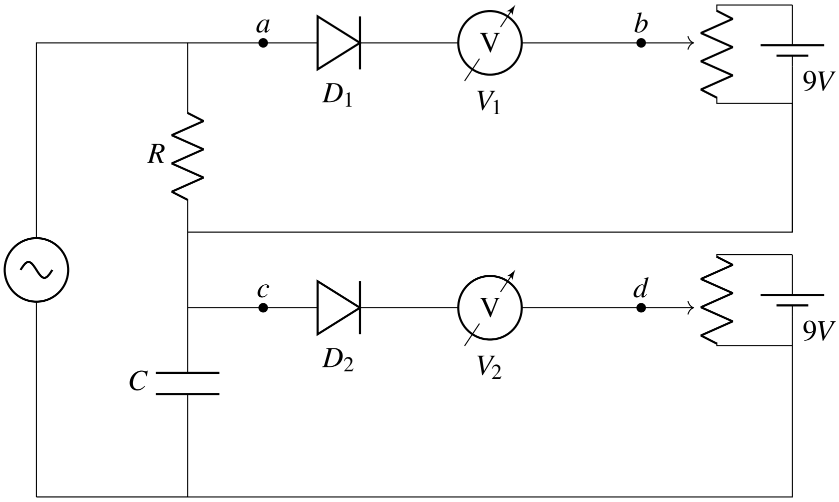



FIGURE 4.13

Looking at the circuit in Fig.4.13 it is obvious that current would flow through the meters $V_1$ and $V_2$ only if the points a and c are at a higher potential as compared to the points b and d. Now suppose you make the potential at b equal to $V_A$ (Fig. 4.12). The meter $V_A$ would, therefore, record a kick only when the voltage at a exceeds $V_A$. Referred to the signal this means the interval A to A' (shown shaded). The problem now reduces to clamping the voltage at point d to $V_B$, the voltage of the other signal at the same instant $t = t_o$. This can be done easily by moving the contact of the second potentiometer until the two meters $V_1$ and $V_2$ start recording the deflection simultaneously at $t = t_o$. You can ascertain this by using the plane mirror strip to superpose one pointer over the image of the other. Note that in case (b) of Fig.4.12 of $V_B$ is negative at $t = t_o$ and hence you will have to reverse the polarity of the battery. Measure the d-c voltages at points b and d. These are the voltages $V_A$ and $V_B$ across R and C at the instant $t = t_o$.

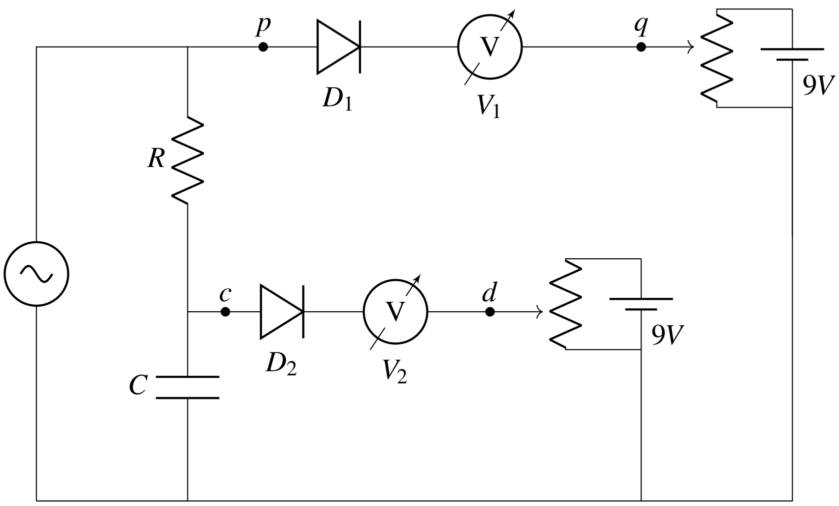



FIGURE 4.14

To measure the instantaneous value of the supply voltage leave one of these, say the one across C, undisturbed and reconnect the other potentiometer as shown in Fig. 4.14. Once again adjust the value of potential at q so as to correspond to the point p (Fig. 4.14). The two meters would again start moving simultaneously. Measure the potential at q to get the value of $V_0$ at t = $t_0$. Now check if the algebraic sum of $V_R$ and$V_C$ is equal to $V_0$. You will now find it is.

The two meters are clamped at t = $t_0$ and hence start recording the kicks simultaneously. Will the deflection in the two last for the same length of time? If not, at what point do the meters clamped at $V_A$ and $V_B$ (Fig. 4.12) stop showing deflection?

Everything is all right and Kirchhoff's law is obeyed if you remember that the latter always implies voltages and currents at a particular instant.

### 4.7.3 Phase difference between $V_C$ and $V_R$

**Experiment IV-C: To determine the phase difference between $V_R$ and $V_C$ in an RC circuit.**

Let Equations (4.8) and (4.9) represent the instantaneous values of voltages across C and R.

$$V_C = V_1 \ cos \ \omega t \tag{4.8}$$

$$V_R = V_2 \ cos \ (\omega t + \phi) \tag{4.9}$$

If you can measure the instantaneous values $V_C$ and $V_R$ at any instant t and the peak values $V_1$ and $V_2$ you can calculate the phase difference $\phi$ between $V_C$ and $V_R$ The instantaneous values can be measured as in Experiment 4-B and the peak values can be either directly read on the meters or measured by any one of the methods given in Section 6. Try different values of C, R and the supply voltage $V_0$.

Similarly determine the phase difference between $V_0$ and $V_R$ and between $V_0$ and $V_C$.

### 4.7.4 Phase difference between $V_C$ and $V_R$ measuring peak voltages

**Experiment IV-D: To study the phase difference between $V_R$ and $V_C$ by measuring the peak voltage values.**

The phase difference between $V_0$, $V_R$ and $V_C$ can also be determined by measuring the peak values alone.
Proceed as follows:

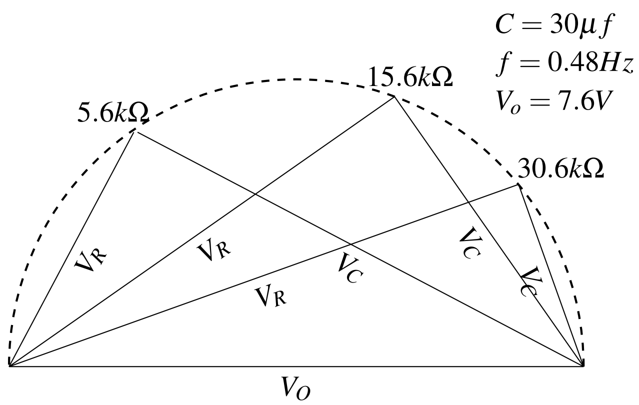



FIGURE 4.15(a)

Keep the output $V_0$ of the oscillator fixed and connect it to a resistor and capacitor in series trying different values of R, C and $\omega$. Measure peak values of the voltages.

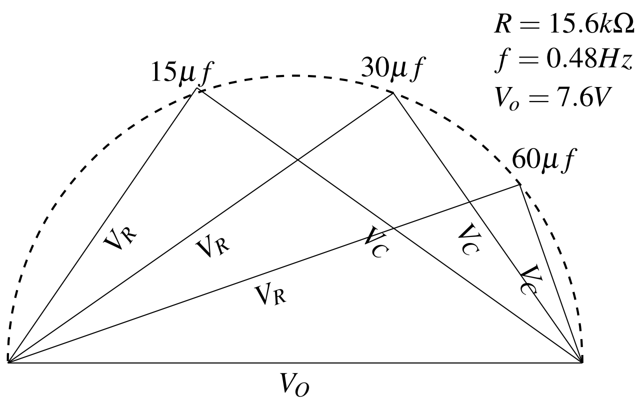



FIGURE 4.15(b)

In each case construct the voltage triangle with the same base always representing the oscillator voltage [Figs. 4.15 (a), (b), (c)]. See if the $V_{RC}$ vertex always falls on a semi-circle with $V_0$ as a diameter. What conclusion would you draw from it about the phase difference between $V_R$ and $V_C$ ?

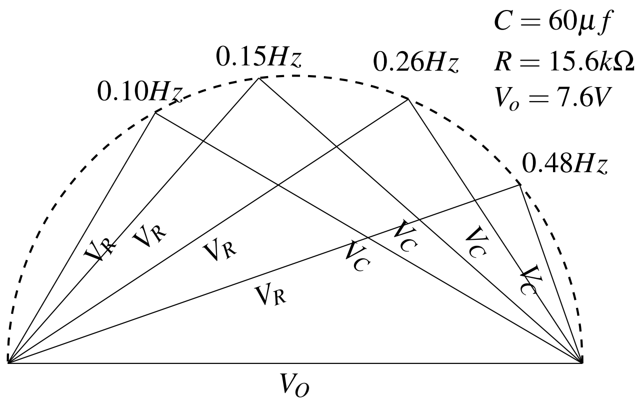



FIGURE 4.15(c)

Usually you would find that the $V_{RC}$ vertex falls a little short of the semi- circle. The phase difference between $V_R$ and $V_C$ would also be a little different from $\pi/2$. Can you explain this qualitatively?

### 4.7.5 Phase relationships of currents

#### Experiment IV-E: To study the phase relationship of currents in different parts of an RC circuit.

The currents in different parts of an RC circuit may not all be in the same phase. These phase differences can be easily determined. Connect the circuit shown in Fig. 4.16, where resistor $R_P$ is in parallel with the capacitor C and another resistor $R_S$ in series with it. Measure the currents (peak values) $I_R$, $I_C$ and $I_0$ by using the three meters (with or without diodes) in the current mode. Construct triangles with sides proportional to $I_R$, $I_C$ and $I_0$. They are called *current triangles*. From them determine the phase relationships. How does the phase difference between $I_R$ and $I_0$ vary as C increases or as $R_P$ increases? For a series of experiments you may set $I_0$ constant and vary $I_R$ and $I_C$ by varying $R_S$ and C. If the triangles are constructed on the common $I_0$ as the base, you would discover that the $I_{RC}$ vertex lies near but not exactly on the semi-circle with $I_0$ as diameter. You should be able to get an idea of the leakage resistance of the capacitor from this.

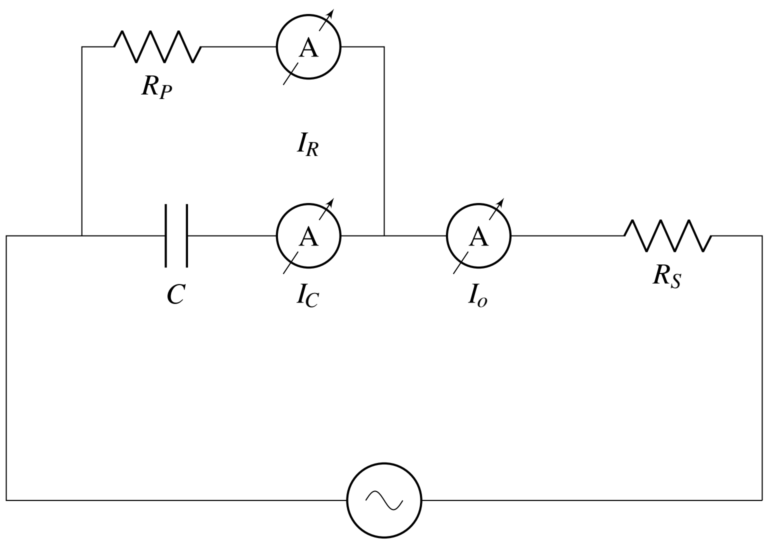



FIGURE 4.16

If you work at a higher frequency and wish to measure the rms values of these currents for constructing the triangle, use the diode bridge with one of the meters (Appendix I-B). Use different pairs of R and C and see how the vertex moves over the semi-circle.

### 4.7.6 RC circuits at different frequencies

**Experiment IV-F: To study the behaviour of an RC circuit at different frequencies.**

i. With a resistor and a capacitor connected in series with the oscillator (Fig. 4.10) construct the voltage triangle as before. Now increase the frequency step by step and see if you can observe the phase relationship of the voltages as in Experiment 4-A. You will see that it gets more and more difficult at higher frequencies. (Why ?) You can, however, use the peak method for measuring the voltages and thus construct the voltage triangles at all frequencies. The shape of the triangle will keep changing, even though you have kept the oscillator voltage and the value of R and C fixed. You will see that the 'vertex' will move on a semi-circle.
ii. Plot a graph between the ratio $V_R/V_C$ and the frequency and see if you get a straight line. What can you infer from the graph?
iii. The current in the circuit is of course,

$$I_o = \frac{V_0}{R} \tag{4.10}$$

The *reactance* of the capacitor is the ratio

$$X_C = \frac{V_C}{I_O} = \frac{V_C}{V_R}R \tag{4.11}$$

From the observed $V_C$, $V_R$ and R calculate $X_C$ at different frequencies and verify if the values agree with the theoretical formula

$$X_C = \frac{1}{2\pi fC} = \frac{1}{\omega C} \tag{4.12}$$

Calculate the capacitance which would give a reactance of 0.2 ohms at a frequency of 0.1 Hz. Does it turn out to be large? Perhaps now you can appreciate why no capacitor has been employed across the current meter for a-c although it has been provided in case of the voltmeter. Remember, this capacitor helps to filter out the a-c component going into the meter.

iv. The current is related to the impressed voltage by the formula

$$I_0 = \frac{V_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \tag{4.13}$$

Note that this relation is very similar to Ohm's law for d-c circuits except that here, in place of the resistance, we have the *impedance* whose magnitude is

$$Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}} \tag{4.14}$$

and both the current and voltage in Equation(4.11) are peak values. (The relation 4.13 remains valid if both $V_0$ and $I_0$ refer to average or rms values),

v. The voltages $V_R$ and $V_C$ are *not* in phase. $V_R$ leads $V_C$ by an angle $\theta$ given by (Appendix IV-B).

$$\theta = \frac{\pi}{2} - \delta \tag{4.15}$$

where

$$tan\ \delta = \omega CR \tag{4.16}$$

Verify this from the triangles constructed for different frequencies.

From Equations (4.15) and (4.16) can you say for what value of R will the phase angle would become $\pi$ /2? What would be the power dissipation in such a circuit? Show this result graphically also. Now compare your results on charging of a capacitor in d-c and a-c circuits. In d-c circuits even when the resistance in the circuit becomes zero there is no escape from the dissipation of energy ($\frac{1}{2}CV^2$),where in purely capacitative circuits in a-c the current is watt less(no energy is spent on charging and discharging ). This is only true after the transients had died out).

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## 4.8 APPENDIX

### 4.8.1 Appendix IV-A ULTRA-LOW FREQUENCY OSCILLATOR

There are a number of ways of generating sinusoidal electrical oscillations in the audio frequency range. Two of the most commonly used are the phase shift and the Wien bridge oscillators.

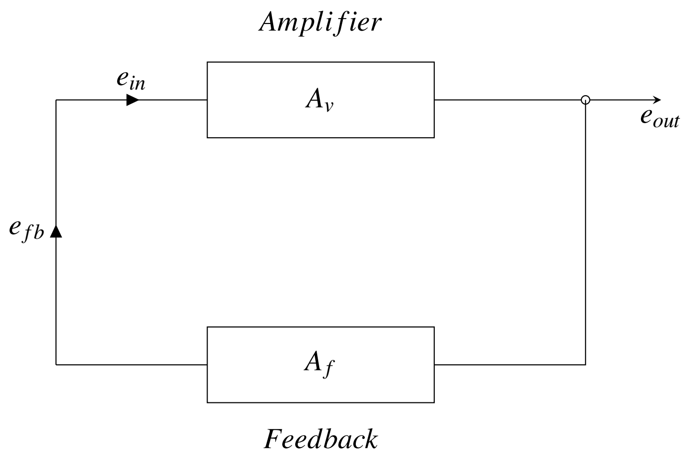



FIGURE IV-A(1) Principle of an oscillator

The basic principle of an oscillator may be understood from the block diagram shown in Fig. IV-A(l). Essentially, one has an amplifier and a *feed back* network. The input signal $e_{in}$ is amplified to give

$$e_{out} = A_v e_{in} \quad (A_v = volttage\ gain)$$

and a portion

$$e_{fb} = A_f e_{out} = A_f A_v e_{in} \quad (A_f = feed\ backfactor)$$

is fed back to the amplifier and, in fact, becomes the input itself, i.e. $e_{fb} = e_{in}$ or, $A_f A_v = 1$. This is known as the Barkhausen condition.

The essence of the condition is that the voltage fed back $e_{fb}$ is again in phase with the input voltage $e_{in}$ so that the combined effect of the amplifier and the feed back network should produce a zero phase shift (or a multiple of $2\pi$). It is possible to arrange that this occurs only for a *selected frequency* so that one gets oscillations at this frequency. One such method is to have a feed back network composed of resistors and capacitors.

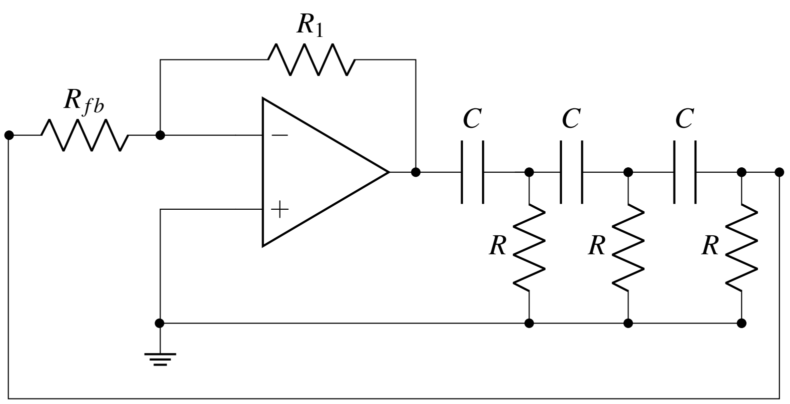



FIGURE IV-A(2) An RC feedback network

Fig.IV-A(2) shows such a circuit. The feed back signal is fed to inverting input of amplifier through $R_{fb}$ indicated and the phase shift is achieved by the three pairs of RC shown.

A complete circuit, with an operational amplifier as an active element and another one as source follower, whose function is essentially to reduce the output impedance (why ?) is shown in Fig.IV-A(3). It will suffice, for understanding the circuit, to think of it as a device which has a high voltage $A_v$ gain, negligible output impedance and a very high input impedance. The oscillator output can be controlled by the $5k\Omega$ potentiometer.

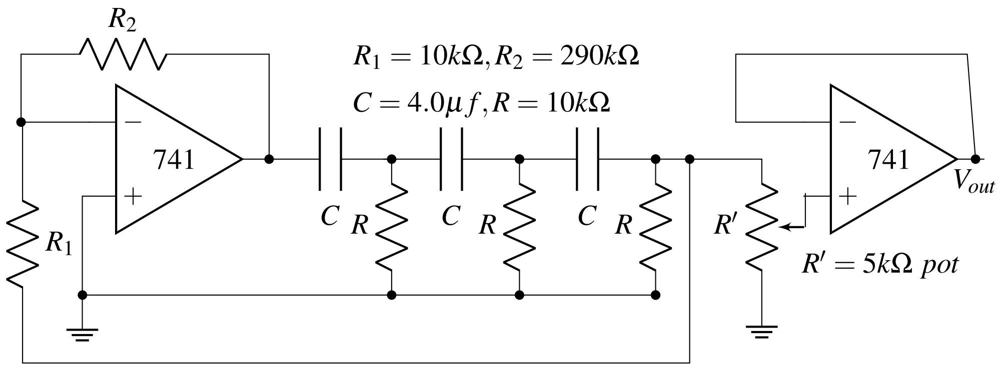



FIGURE IV-A(3) A complete circuit of a phase shift oscillator

This circuit has been successfully operated but has a number of disadvantages. It takes a long time, of the order of minutes, for the transients to die out and to vary the frequency one requires simultaneous change of all three resistors.

The Wien bridge oscillator utilizes a balanced bridge as the feed back network [Fig. IV-A(4)].

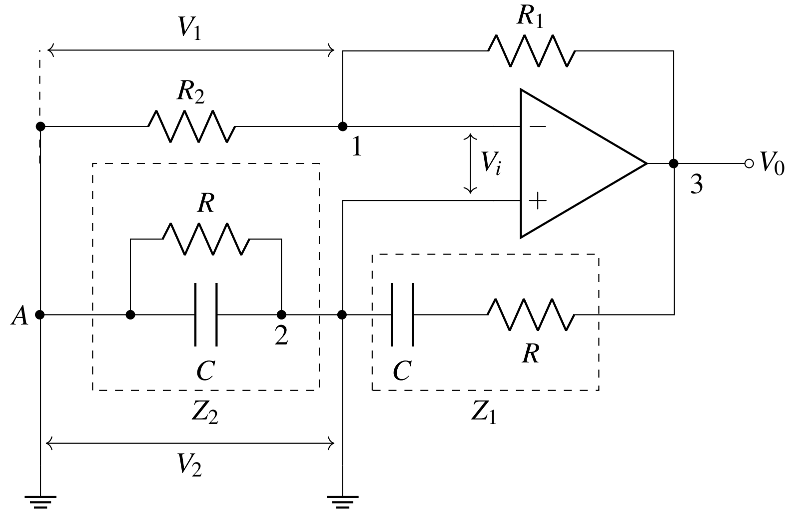



FIGURE IV-A(4) A wien bridge oscillator

An operational amplifier is used as the active element in this circuit. The voltage fed back to the operational amplifier is the difference $V_2 - V_1$ and the feed back factor $A_f$ can be shown to be

$$A_f = \frac{(V_2 - V_1)}{A_{out}} = \frac{Z_2}{Z_1 + Z_2} - \frac{R_2}{R_1 + R_2}$$

Here, $Z_1$ is the impedance in the arm $2-3$ and $Z_2$ in the arm $2-A$. Writing

$$Z_1 = R + \frac{1}{i\omega C} \quad and \quad Z_2 = \frac{R}{1+\frac{1}{i\omega CR}}$$

we obtain, after some algebra,

$$A_f = \left( \frac{R}{3R - \frac{i}{\omega C}(1-\omega^2 C^2 R^2)} \right) - \frac{R_2}{R_1+R_2} \tag{IV-A-1}$$

Clearly, $A_f$ will be real** if $1-\omega^2 R^2 C^2 = 0$, i.e. the frequency is

$$f_0 = \frac{\omega}{2\pi} = \frac{1}{2\pi RC}$$

The bridge is adjusted so that at this frequency $A_f$ is slightly positive; i.e. $\frac{R_2}{R_1+R_2}$ is smaller than $1/3$.

The condition of setting the loop gain $A_f A_v = 1$ is, in practice, not quite good enough, because if for some reason the value of $A_f A_v$ drops below unity, the input becomes smaller and smaller and the oscillator ceases to function. Therefore, it is necessary to make the loop gain slightly larger than 1. The oscillations will grow now in the initial stages, but this does not happen indefinitely due to the onset of non-linearity of operation in the amplifier.

A practical consideration is that the amplitude of the output of the oscillator must be stable even if there are fluctuations in the amplifier and other components. This is achieved by making either $R_1$ or $R_2$ a non-linear device. For example,$R_2$ may be an element whose resistance increases with temperature which in turn, is determined by the (rms) current through it. Suppose the amplification $A_v$ decreases. The current and hence the value of $R_2$ will both decrease and as a result the factor $A_f$[see Equation IV-A-1] increases. Thus the product $A_v A_f$ is maintained roughly constant.

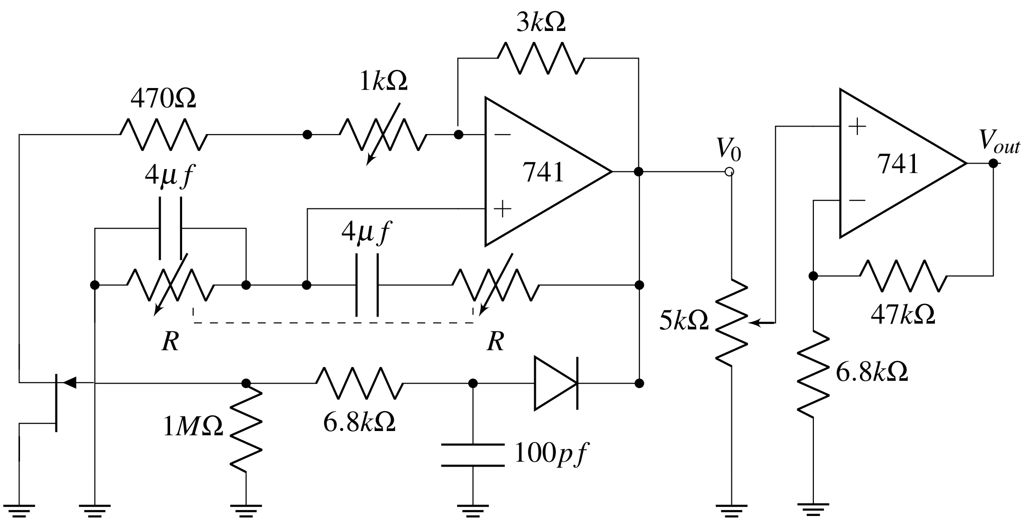



FIGURE IV-A(5) A complete circuit of the ultra-low frequency oscillator

An important factor is that the thermal response time of such a device should be much larger than the period of the oscillations so that the resistance is very nearly constant over a single cycle. This poses a problem at the very low frequency (0.1 hz) that is desired here since a simple incandescent lamp does not have such a long response of changing its resistance.

**Real $A_f$ means that the feedback is in phase

It is for this reason that the circuit has had to be modified using an FET(BFW 10) which achieves such a large response time. (For details of the principle of FET and its operation, see Millman and Halkias, Integrated Electronics, Chapter 10).

The oscillator [Fig.IV-A(5)] is operated using a $\pm$ 12 *Volt* regulated power-supply. It gives an output of 20*Volt* peak to peak and can drive a $1k\Omega$ load. The unit can be short circuited without any damage (although the oscillator is not very useful, so short circuited !). Several frequencies in the interval 0.1 hz to 10 hz are available for use.

## 4.8.2 Appendix IV-B ALTERNATING EMF, RC CIRCUIT AND ITS ANALYSIS

### 4.8.2.1 Root Mean Square values and Power

Any a-c variation, as for example a voltage, can be represented as

$$V = V_0 \; cos\; (\omega t + \phi) \tag{IV-B-1}$$

The period is $T = \frac{2\pi}{\omega} = \frac{1}{\nu}$, here $\nu$ is the frequency.

The average value of V over a cycle (or, for that matter, over a number of cycles) is zero, i.e.

$$\overline{V} = \frac{1}{T}\int V dt = \frac{1}{T}\int_0^T V_0 \; cos\; (\omega t + \phi)\, dt$$

=0

On the other hand, the average of the square of the quantity is not zero:

$$\overline{V^2} = \frac{1}{T}\int_0^T V_0^2 \; cos^2\; (\omega t + \phi)\, dt = \frac{1}{2}V_0^2$$

Thus the root mean square source of the voltage, designated $V_{rms}$ is given by

$$V_{rms} = \frac{V_0^2}{\sqrt{2}} \tag{IV-B-2}$$

$V_0$ being the peak value.

An exactly similar relation holds for the current,

$$I_{rms} = \frac{I_0}{\sqrt{2}} \tag{IV-B-3}$$

The mean power dissipated by an a-c source is given by

$$\overline{P} = \frac{1}{T}\int_0^T VI = V_{rms}\, I_{rms} \; cos\; \theta$$

where $\theta$ is the phase difference between V and I. The proof is simple:

$$\overline{P} = \frac{1}{T}\int_0^T V_0 \; cos\; (\omega t + \phi)\; I_o \; cos\; (\omega t + \phi + \theta)$$

$$\overline{P} = \frac{V_0 I_0}{2\pi}\int_0^T cos\; (\tau)\; cos\; (\tau + \theta)\, dt$$

where we have put $\omega t = \tau$. Integration leads to

$$\overline{P} = \frac{V_0\, I_0}{2\pi}\, cos\ \theta = V_{rms}.I_{rms}\ cos\ \theta$$

Notice the important term cos $\theta$, Thus, if $\theta = \pi/2$ there is no power dissipation, on average.

This factor cos $\theta$ is called the *power factor* of the circuit.

#### 4.8.2.2 RC Circuit with a-c Voltage

The emf equation for an RC circuit is

$$\frac{q}{C} + IR = V_0\ cos\ (\omega t) \qquad \text{(IV-B-4)}$$

where q is the charge on the capacitor, I is the current in the circuit.The phase of the voltage is taken as zero so that other phases may be measured relative to it.

We shall discuss only the steady state solution (transients will be discussed in Appendix IV-C). This solution can be put either in terms of the current or the charge. We try

$$I = I_0\ cos\ (\omega t + \phi) \qquad \text{(IV-B-5)}$$

so that

$$q = \int I dt = \frac{I_0}{\omega}\ sin\ (\omega t + \phi) \qquad \text{(IV-B-6)}$$

Substituting in Equation (IV-B-4) we obtain

$$\frac{I_0}{\omega C}\ sin\ (\omega t + \phi) + I_0 R.\ cos\ (\omega t + \phi) = V_0\ cos\ (\omega t + \phi)\ cos\ \phi +\ sin\ (\omega t + \phi)\ sin\ \phi \qquad \text{(IV-B-7)}$$

Since this relation is to hold for all time, we equate the coefficients of $cos\ (\omega t + \phi)$ and $sin\ (\omega t + \phi)$ separately. This gives :

$$V_0\ sin\ \phi = \frac{I_0}{\omega c}$$

$$V_0\ cos\ \phi = I_0.R$$

Hence

$$I_0 = \frac{V_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \qquad \text{(IV-B-8)}$$

and

$$tan\ \phi = \frac{1}{\omega CR} \qquad \text{(IV-B-9)}$$

The voltage across the capacitor is q/C. Then from (IV-B-6)

$$V_0 = \frac{q}{C} = \frac{I_0}{\omega C}\ cos\ (\omega t + \phi - \pi/2) \qquad \text{(IV-B-10)}$$

These equations tells us the following:

a) The current I *leads* the applied voltage signal V by $\phi$, where $tan\ \phi = \frac{1}{\omega RC}$
b) The current I *leads* the capacitor voltage $V_C$ by $\pi/2$
c) The voltage $V_C$ across the capacitor *lags* behind the applied signal V by $\pi/2 - \phi = \delta$ , where $\delta = \omega CR$
d) The *impedance* of the circuit, defined by $\frac{V_0}{I_0}$ which equals $\frac{V_{rms}}{I_{rms}}$ is given by

$$|Z| = \frac{V_0}{I_0} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}} \tag{IV-B-11}$$

The phase difference between V and I gives the power factor $cos\phi$. From Equation (IV-B-9) and (IV-B-11)

$$cos\ \phi = \frac{\omega CR}{\sqrt{1+\omega^2 C^2 R^2}} = \frac{R}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} = \frac{R}{|Z|} \tag{IV-B-12}$$

#### 4.8.2.3 Representation by Complex Quantities

We have earlier mentioned the use of vector (or phasor) diagrams to take care of both magnitude and phase in the superposition of different voltages or currents in an a-c circuit. That method cannot show us how the phase differences originate. We shall now outline an algebraic method of analysis which uses complex quantities.

Any complex number can be written as

$$Re^{i\theta} = R\ cos\ \theta + iR\ sin\ \theta$$

where R is called the *modulus* (or magnitude) and $\theta$ the argument (or phase). $R\ cos\ \theta$ is the real part and $R\ sin\ \theta$ the imaginary part. For a voltage signal $V_0\ cos\ \omega t$ , we now decide to mix the imaginary part $iV_0\ sin\ \omega t$ and write the the complex signal in Equation (IV-B-4)

$$V = V_0 e^{i\omega t} \tag{IV-B-13}$$

Similarly in Equation (IV-B-5) let us consider the complex current to be

$$I = I_0 e^{i\omega t + \phi} \tag{IV-B-14}$$

The actual voltage and current are the real parts of these V and I respectively. But mixing the imaginary parts makes the analysis simpler, as you will see.

Equations (IV-B-4) to (IV-B-6) now become

$$\frac{q}{C} + IR = V_0\ e^{i\omega t}$$

$$I = I_0\ e^{i\omega t + \phi}$$

$$q = \frac{I_0}{i\omega}\ e^{i(\omega + \phi)}$$

and Equation(IV-B-7) takes the simple form

$$I_0 \left( R + \frac{1}{i\omega C} \right) = V_0\ e^{-i\phi}$$

The impedance Z, defined by $\frac{V}{I}$ as distinct from $\frac{V_0}{I_0}$, which gave modulus $|Z|$ is given by Equations (IV-B-13) and (IV-B-14) as

$$Z = \frac{V_0}{I_0} = |Z|\ e^{-i\phi}$$

Thus the impedance of the circuit has not only magnitude $|Z|$ but also a phase, which is -$\phi$ here. Equation(IV-B-15) now gives

$$Z = R + \frac{1}{i\omega C} = R - i\frac{1}{\omega C} \tag{IV-B-16}$$

Thus the impedance has two components; the part associated with the resistor is real(R), whereas the part associated with the capacitor is imaginary $(\frac{-i}{\omega C})$. Generalizing this, we may consider the case of R and C connected in parallel to source of angular frequency $\omega$. The impedance will be

$$Z = \frac{R\frac{1}{i\omega C}}{R + \frac{1}{i\omega C}} = \frac{R}{1+\omega^2 C^2 R} - i\frac{\omega C R^2}{1+\omega^2 C^2 R^2}$$

In general, for any network the impedance Z will have a real and an imaginary part and it can be written as

$$Z = |Z|\ e^{i\theta} \tag{IV-B-17}$$

Now if V is written in complex form,the current I can be written as

$$I = \frac{V}{Z} = \frac{V_0\ e^{i\omega t}}{|Z|\ e^{i\theta}} = I_0\ e^{i(\omega t-\theta)} \tag{IV-B-18}$$

Thus, while $V_0/|Z|$ gives us only amplitude $I_0$, $V/Z$ gives the amplitude as well as phase of the current. The $\theta$ in the expression for z is called the *impedance*.

The actual current is the real part of te expression in Equation (IV-B-18)

Let us apply this method to the series RC circuit. from Equation (IV-B-16),

$$Z = R - i\frac{1}{\omega C} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}\ e^{i\theta}$$

where

$$tan\ \theta = \frac{-(\frac{1}{\omega C})}{R} = \frac{-1}{\omega C R}$$

The current for the voltage $V_0\ cos\ \omega t$ is, therefore,

$$I = Re\left[\frac{V_0\ e^{i\omega t}}{\sqrt{R^2+\frac{1}{\omega^2 C^2}}\ e^{i\theta}}\right] = \frac{V_0}{\sqrt{R^2+\frac{1}{\omega^2 C^2}}}\ cos\ (\omega t - \theta)$$

which is same as given by Equations(IV-B-5) and (IV-B-8), since $\theta$ here equals -$\phi$ there.

## 4.8.3 Appendix IV-C TRANSIENTS

### 4.8.3.1 Transients

We had discussed, in Appendix IV-B, the behaviour of circuits under an applied ac voltage and shown how graphical methods may be used to obtain peak or rms values of currents and voltages. These solutions, however, are only the steady state solutions. In general, the response of the circuit involves both a *transient*[††] term and a steady state term. The transient term is important only in the early stages of switching on the a-c voltage i.e. within a few cycles. Let us see how it comes about.

The equation for charging in an RC circuit is

$$IR + \frac{q}{C} = V(t) \qquad \text{(IV-C(1))}$$

where $V(t)$ is the applied voltage as a function of time, q the charge on the capacitor and I, the current in the circuit. Writing $I = \frac{dq}{dt}$ we have

$$\frac{dq}{dt} + \frac{q}{RC} = \frac{V(t)}{R} \qquad \text{( IV-C(2))}$$

an equation which can be solved provided we know the form of the applied voltage V(t).

In Appendix II-B, we have discussed the solution for a particular case, namely,

$$\begin{aligned} V(t) &= 0 \quad \text{for } t < 0 \\ &= V_0 \quad \text{for } t > 0 \end{aligned} \qquad \text{(IV-C(3))}$$

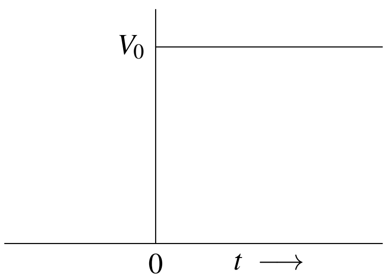
This corresponds to turning on a d-c source at t=0. We recall that the solution is (subject to the condition that the capacitor is initially uncharged)

$$\begin{aligned} V_C(t) &= V_0\,(1 - e^{-t/RC}) \\ I(t) &= I_0\, e^{-t/RC} \end{aligned} \qquad \text{(IV-C(4))}$$

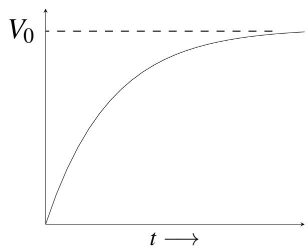
where $V_C$ is the voltage on the capacitor, I the current and $I_0 = V_0/R$ is the maximum current which occurs at t=0. Fig. IV-C(1) displays the result graphically.

Consider now the case when the input is an a-c voltage for t > 0, i.e.

$$\begin{aligned} V(t) &= 0 \qquad\quad t < 0 \\ &= V_0 \, cos\, \omega t \quad t > 0 \end{aligned} \qquad \text{(IV-C(5))}$$




$(a)$ *Input Voltage*




$(b)$ *Capacitor voltage*

FIGURE IV-C(1)

[††]The dictionary meaning of 'transient' is *momentary* or of *short duration*

The complete solution [‡‡] for the capacitor voltage can be shown to be

$$V_C(t) = \frac{q(t)}{C} = \frac{V_0}{\sqrt{1+\omega^2R^2C^2}}\left[cos\ (\omega t - \alpha) - \frac{1}{\sqrt{1+\omega^2R^2C^2}}e^{-t/RC}\right] \tag{IV-C(6)}$$

where

$$tan\ \alpha = \omega RC$$

$$cos\ \alpha = \frac{1}{\sqrt{1+\omega^2R^2C^2}}$$

This could also be written as

$$V_C(t) = V_0\ cos\ \alpha\ [cos\ (\omega t -\ \alpha) - e^{-t/RC}\ cos\ \alpha] \tag{IV-C(7)}$$

There are two terms in Equation (IV-C-7) and they have entirely different behaviour with time. The first term represents a voltage that oscillates with precisely the applied frequency but lags behind in phase by $\alpha$. The second term, on the other hand, is appreciable only for $t \leq RC$ and for larger times it quickly dies out. This exponentially decaying term is called a *transient* term and the first term, the steady state term[§§]. If we want the circuit behaviour to determined essentially by the steady state term, we ought to wait for $t \gg RC$; in practice, times of the order of 5 RC ought to suffice.

The transient term arises essentially because of our initial act of switching on the voltage at t=0. If we set $\omega = 0$ in Equation (IV-C(6)), we recover the d-c response mentioned earlier. Without the transient term this would not have been the case. In fact, in a sense, the entire behaviour of the circuit after switching on a d-c voltage is a transient behaviour. The effect of this transient term, as far as the experiment is concerned, is to delay the setting up of a steady voltage and current in the circuit. This can be easily ascertained by looking at a meter which, it will be found, does not start recording a steady deflection immediately on switching on the circuit but does so only after a few cycles. If you employ an ultra-low frequency source, as you do in experiments described in this chapter, you will observe that the needle of the meter swings back and forth and with every swing the deflection goes on increasing until it reaches a steady value.

[‡‡] You should verify this by substitution. The initial condition is $V_C(0) = 0$

[§§] Compare this term with the equation (IV-B(10), remembering that $\alpha = \pi/2 - \phi$ and $V_0/I_0$ is given by Equation (IV-B(8))

V

# CHAPTER 5

# 5. Study of RC Circuit with a-c Mains

## 5.1 Introduction

In chapters 3 and 4 we have done a detailed analysis of the RC circuit using sources of varying and sinusoidal emf. We learnt how to measure the peak, average and even instantaneous values of currents and voltages in these circuits. We also learnt how to represent these quantities on vector diagrams and measure phase differences.

In this chapter we shall study the behaviour of RC circuit using the a-c mains. In fact, since you do not need the high mains voltage (220 volts) you will use a supply (transformer) which gives lower a-c voltages at a frequency of 50 hz. As pointed out in the preceding chapter it is not possible to make measurements at the frequency of 50 hz with d-c meters. The experiments that follow are, therefore, performed measuring rms values of current and voltage (or their peak values) with suitably converted d-c meters (Appendix I-B). The visual effects observed in the study of an RC circuit with an ultra-low frequency source (Chapter 4) are, naturally, lost in these experiments but this should not render the understanding of the phenomena impossible. A thorough reading of the discussions given in that chapter should, however, prove very useful in understanding these experiments which are essentially similar to those described earlier. You should also clearly understand why it is not possible to measure instantaneous values at high frequencies and the function of the bridge-rectifier in measurement of rms values (Appendix I-B).

## 5.2 The Network Board - 5

A schematic arrangement of various components provided on this board is given in Fig. 5.1. As you can see from it there are a number of resistors and capacitors on the board. A transformer with tappings for voltages from 20 to 100 volts, capable of supplying a maximum current of 100 mA, is provided to serve as a-c source. Two d-c meters along with bridge-rectifier circuits are used to measure a-c voltage and current. The meters are calibrated to measure rms values.

In drawing vector diagrams, you may simply represent all vectors by their rms rather than peak values—i.e. the lengths are taken proportional to the rms values. Since the peak value of current or voltage is simply $\sqrt{2}$ times the rms value, you can calculate either one whenever necessary.

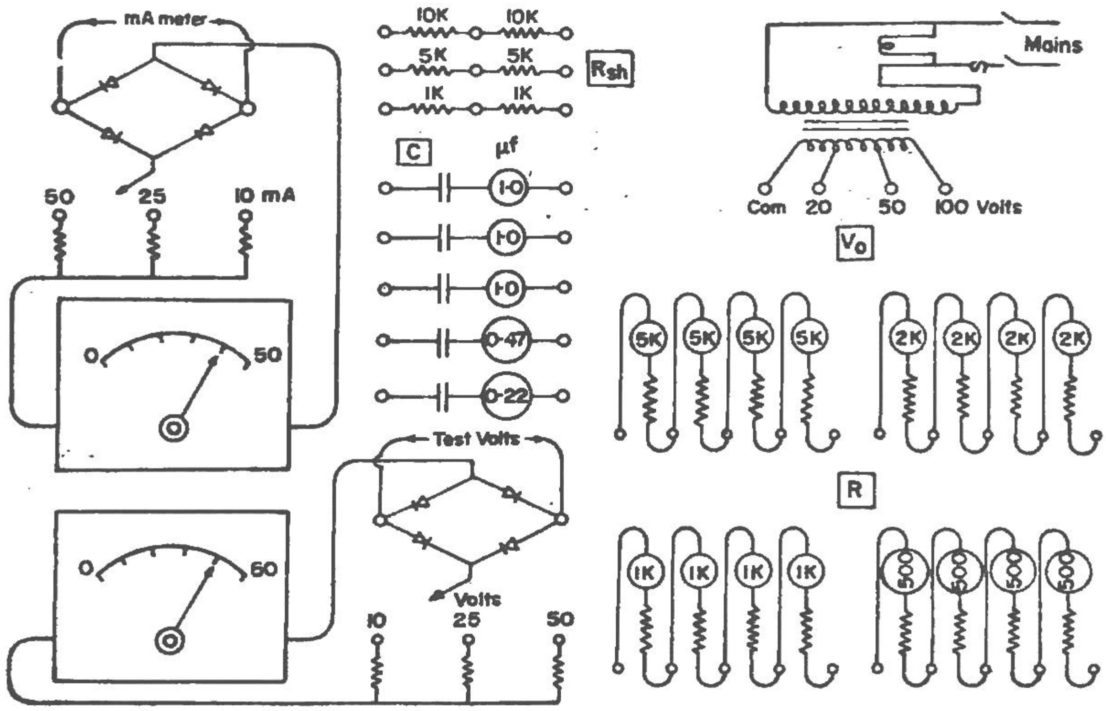



Figure 5.1: FIGURE 5.1 Network Board 5

## 5.3 Experiments

### 5.3.1 Purely resistive circuit

#### Experiment V-A: To study a purely resistive circuit.

Connect a set of resistors and the current meter in series to the transformer and verify that the relation

$$\frac{V_1}{R_1} = \frac{V_2}{R_2} = \frac{V3}{R_3} = \ldots = I \tag{5.1}$$

holds, where $V_1$, $V_2$ etc. are the rms voltages across $R_1$, $R_2$ etc and I, the current in the circuit. The experiment is *trivial* except that it reminds you clearly that for resistors in series the result is the same for a-c as for d-c. Draw a vector diagram for all the voltages in the circuit.

### 5.3.2 Simple RC Circuit

#### Experiment V-B: To study a simple RC circuit.

A capacitor C and a resistor R are connected in series to the source along with the current meter. Measure I, $V_C$, $V_R$ and $V_{CR}$, the last one being the voltage across R and C together.

Verify the relations

$$V_R = IR \text{ and } V_C = \frac{I}{\omega C} \tag{5.1}$$

where $\omega$ is the angular frequency (i.e. $2\pi$ times the frequency f of the a-c supply). From Equation (5.1) can you say what would happen to the relative values of $V_R$ and $V_C$ at very low and very high frequencies?

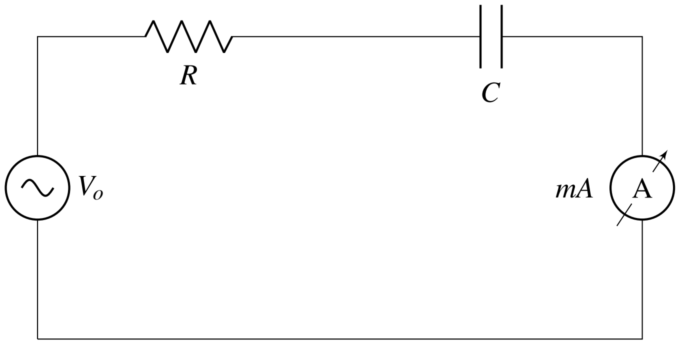



FIGURE 5.2

Use different values of C and adjust R in each case to obtain the same current. Draw a graph between $V_C$ and $1/C$.

### 5.3.3 Vectorial addition of voltages

#### Experiment V-C: To study the vectorial addition of voltages across capacitor and resistor in an RC circuit.

Connect some capacitors and some resistors in series to the source (Fig. 4.11). Establish the relation

$$V_{C_1}\ \omega C_1 = V_{C_2}\ \omega C_2 = \ldots = \frac{V_{R_1}}{R_1} = \frac{V_{R_2}}{R_2} = \ldots = I \tag{5.2}$$

As far as rms values of currents and voltages are concerned, the impedance $1/\omega C$ plays the same role as the resistance R.

Notice that the voltages $V_R$ and $V_C$ do not add algebraically. For example, for just one capacitor and one resistor in series,

$$V_R + V_C > V_0$$

($V_0$ being the source voltage). Do you see that

$$V_R^2 + V_C^2 = V_0^2$$

Explain this on a vector diagram. Also check for a series of resistors and capacitors that

$$V_{R_1} + V_{R_2} + V_{R_3} = V_{R_{123}}$$

and

$$V_{C_1} + V_{C_2} + V_{C_3} = V_{C_{123}}$$

where $V_{R_{123}}$ is the voltage across all the three resistors in series and $V_{C_{123}}$ the voltage across all the three capacitors.

### 5.3.4 Behaviour of an actual capacitor

#### Experiment V-D: To study the deviation in behaviour of an actual capacitor from an ideal one.

With a constant source voltage $V_0$, connect a resistor and a capacitor in series and measure $V_R$ and $V_C$ for different values of R and C (keeping source voltage fixed), construct the vector triangles of sides $V_R$, $V_C$ and $V_0$; the best way is to draw all the triangles with the same base $V_0$. Then, for different values of $V_R$ and $V_C$, we ought to obtain right-angled triangles with $V_0$ as hypotenuse. Thus, the vertices of the triangles ought to be on a semi-circle. In reality it may not be so. Can you guess the reason ?

### 5.3.5 Leakage representation by series resistance

#### Experiment V-E: To represent the deviation in the behaviour of an actual capacitor by a series resistance.

From the foregoing experiment you have probably learnt the difference between an ideal and an actual capacitor. The latter always has some leakage across it which can be represented by a leakage resistance, in series or in parallel with it. This would imply that its reactance is not purely capacitive. This and the next experiment will help you to understand these remarks better.

Connect a capacitor and a number of equal resistors in series with the source [Fig.5.3(a)]. Measure the potential differences in pairs $V_{C_o}$ and $V_{14}$, $V_{C_1}$ and $V_{14}$, $V_{C_2}$ and $V_{24}$, $V_{C_3}$ and $V_{34}$. With a source voltage $V_0$ as base construct triangles for each of these pairs [Fig. 5.3(b)]

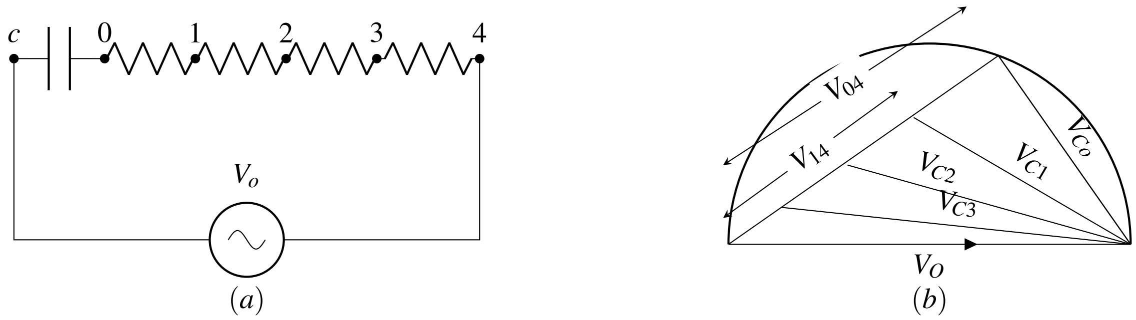



FIGURE 5.3

Determine (from the diagram) the phase difference between constituents of each of the above pairs, for example, between $V_{C_o}$ and $V_{C_4}$ and so on. You will find that whereas phase difference between $V_{C_o}$ and $V_{C_4}$ is almost $\pi/2$ with the vertex of the triangle nearly touching the semi-circle, the same is not true of the pairs $V_{C_1}$ and $V_{14}$; $V_{C_2}$ and $V_{24}$ etc. The phase difference in these cases is quite different from $\pi/2$ and the vertices lie well inside the semi-circle. Thus, when you measure $V_{C_1}$, $V_{C_2}$ etc. what you are effectively doing is to associate some resistance with the capacitor and then measure the voltage across the combination which makes the vertex move inwards. This is an *exaggerated* picture of an actual capacitor. In this sense, you can represent the behaviour of a non ideal capacitor by treating it as a combination of a small series resistance and an ideal 'capacitor of a slightly higher capacitance. (see Experiment 5-H).

### 5.3.6 Represent leakage by a shunt

#### Experiment V-F: To represent the leakage resistance of a capacitor by a shunt.

Connect a resistor R in parallel with a capacitor C (Fig. 5.4a) and then this combination in series with another resistor $R_o$ and the source of power.

Measure $V_{CR}$, $V_{R_o}$ and $V_0$ (the source voltage). Draw a vector diagram for these voltages(Fig. 5.4b. You will notice that the phase difference between $V_{CR}$ and $V_{R_o}$ is not $\pi/2$. By varying R you will also notice that the phase difference approaches $\pi/2$ as R is increased.

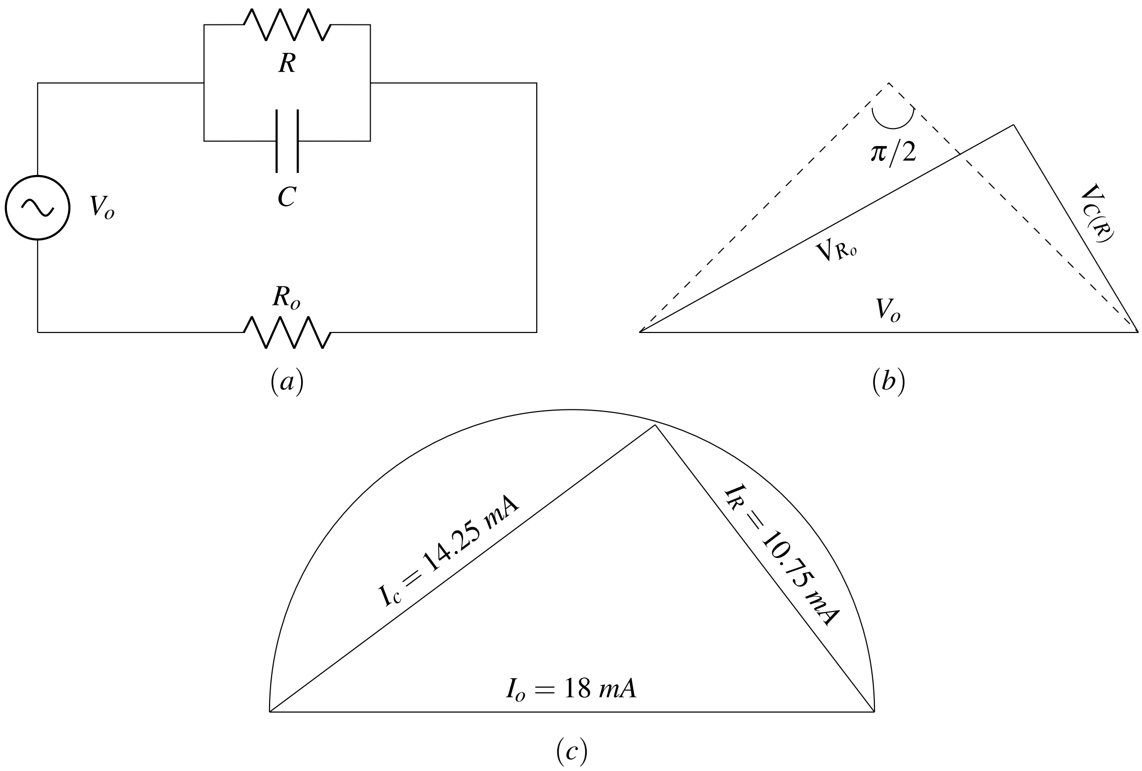



FIGURE 5.4

In fact, if you remove the shunt R and draw the vector diagram afresh (dotted) you will find the phase difference to be nearly $\pi/2$. This means that a real capacitor (as opposed to an ideal one) can also be treated as having a large resistance as a shunt with it. However, the capacitors you have been using are not very far from ideal at least under the conditions used.

### 5.3.7 Quadrature of currents in R and C arms

#### Experiment V-G: To verify that the currents in resistive and capacitive arms of an RC circuit are in quadrature.

In the arrangement of Fig. 5.4(a), measure the current in the three arms R, C and $R_o$. Construct a current triangle to represent these, taking $I_{R_o}$ as base. Verify that $I_R$ and $I_C$ have a phase difference of $\pi$ /2. [see Fig. 5.4(c)]. Repeat this for different values of the components.

### 5.3.8 Equivalent series resistance for a shunt across C

#### Experiment V-H: To determine the effective series resistance for a capacitor corresponding to a shunt across it.

Connect a resistor R in parallel to a capacitor and another $R_o$ in series as shown [Fig. 5.5(a)]. Measure the voltages $V_{CR}$, $V_{R_o}$ and $V_o$ and construct the voltage triangle [Fig. 5.5(c)]. The phase difference between $V_{CR}$ and $V_{R_o}$ is not $\pi/2$. Extend the $V_{R_o}$ line to meet the semicircle with $V_o$ as diameter. This gives from which you deduce $R'$ and $C'$. Thus you get the effective resistance $R'$ and capacitance $C'$ which, when connected in series [Fig. 5.5(b)], give the same impedance as C shunted with R.

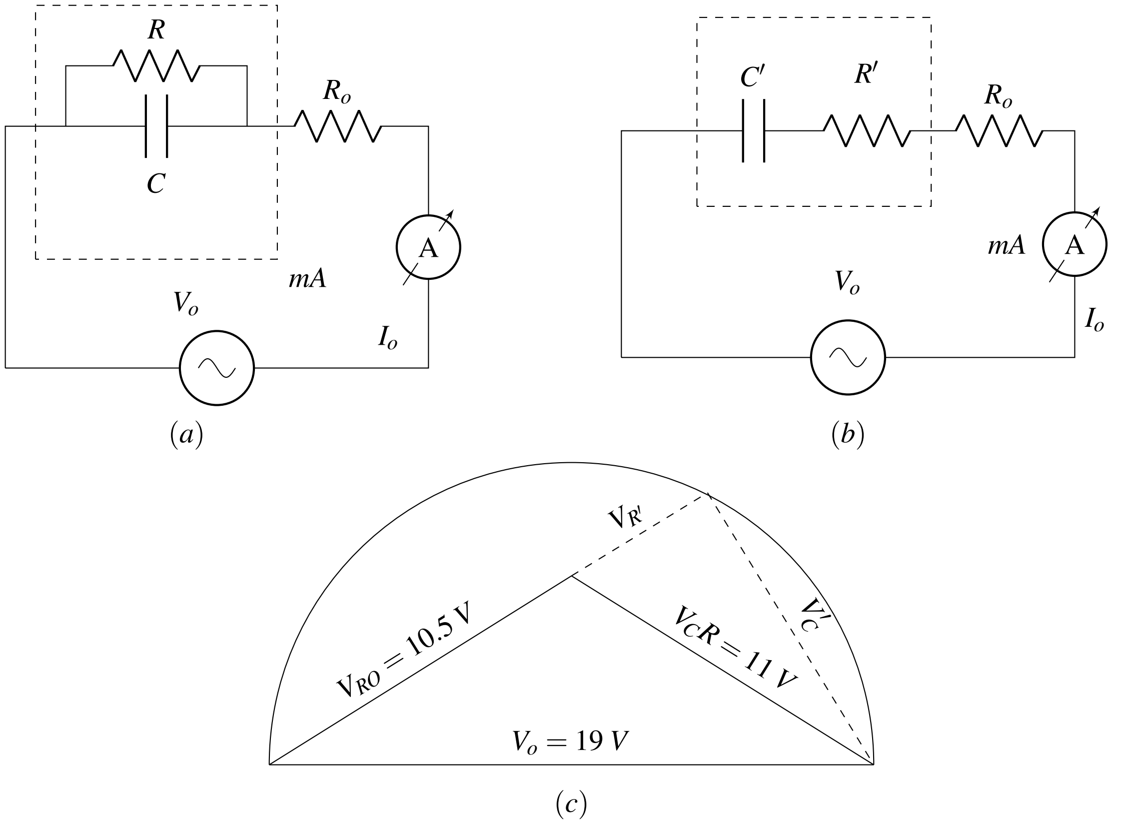



FIGURE 5.5

Try this for different values of R. The power loss resistance $R'$ is given by the relation *

$$\frac{V_C^2}{R} = I_0{}^2 R'$$

where $V_C$ is the voltage across C [Fig. 5.5(a)] and $I_o$ is the rms value of current (delivered by the source)

Now you should be able to appreciate better why the leakage resistance of a capacitor can be represented by a series resistance as well as a shunt (Experiments 5-E and 5-F).

*In the above experiment we have seen how, the energy loss in a real capacitor can be represented by an equivalent series resistance R so that $I^2R$ gives the power loss, I being the rms current. The factor*

*Comparing Figs. 5.5(a) and 5.5(b) the equality of impedance means

$$\frac{R\left(\frac{1}{i\omega C}\right)}{R+\frac{1}{i\omega C}} = R' + \frac{1}{i\omega C}$$

Equating real and imaginary parts separately gives

$$R' = \frac{R}{1+\omega^2R^2C^2} \approx \frac{1}{\omega^2C^2R}$$

$$C' = C(1+\frac{1}{\omega^2C^2R^2)} \approx C$$

since the leakage resistance R is much greater than $1/\omega C$.Hence

$$I_0^2R' = \frac{I_0^2}{\omega^2C^2R} = \frac{V_C^2}{R}$$

$$cos\ \phi = \frac{R}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

*is called the* power factor *of the capacitor. ( For R=0, it is 0). Since usually* $R \ll \frac{1}{\omega C}$*, we may write (see Appendix IV-B)*

$$cos\ \phi = \omega CR$$

*A 1 $\mu f$ capacitor operated at 50 hz has a power factor of 0.001. What would be the equivalent series resistance?*
*Suppose the leakage is to be represented by a resistor in parallel with the capacitor, what would be the equivalent resistance? (This is called the equivalent shunt resistance). Repeat the above calculations for a frequency of 1 khz. At low frequencies the power factor remains constant.*

## 5.3.9 Impedance in a series RC circuit

### Experiment V-I: To measure the impedance in a simple RC circuit.

Taking different combinations of R and C in series, measure the current $I_o$ by a current meter or by measuring the voltage drop across a small test resistance (how small should it be?) in series.

Thus measure the impedance (see Appendix IV-B)

$$|Z| = \frac{V_0}{I_0}$$

and verify the relation

$$|Z| = \sqrt{R^2 + \frac{1}{\omega^2 C^2}} \tag{5.3}$$

## 5.3.10 Impedance in a parallel RC circuit

### Experiment V-J: To measure the impedance in a simple parallel RC circuit.

Make different combinations of R and C in parallel and measure the impedance in the same manner.

$$\frac{1}{|Z|^2} = \frac{1}{R^2} + \frac{1}{\frac{1}{\omega^2 C^2}} \quad or \quad |Z| = \sqrt{R^2 + \frac{1}{\omega^2 C^2}} \tag{5.4}$$

## 5.3.11 Equivalent circuits for complex RC networks

### Experiment V-K: To represent complex RC circuits by equivalent simple ones.

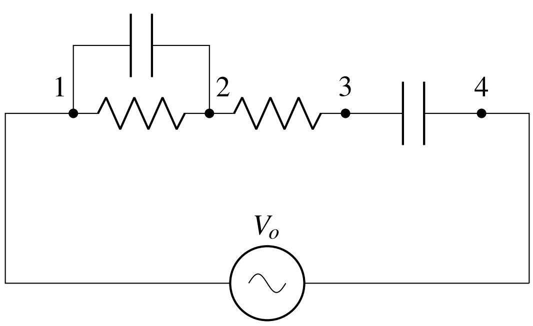



FIGURE 5.6

Make up a 'hybrid' combination of resistors and capacitors (i.e. some in series and some in parallel as shown in Fig. 5.6). Measure the impedance. Now represent the circuit by an equivalent circuit with R and C in series and another with R and C in parallel.

*Suppose in the circuit of Fig. 5.6, the impedance of each of the capacitors and resistors is the same. Draw a vector diagram to represent the voltges* $V_{12}$, $V_{13}$ *and* $V_{14}$. *(First just attempt a qualitative answer . Then try to be precise)*

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## 5.4 APPENDIX

### 5.4.1 Appendix V-A AUDIO-FREQUENCY OSCILLATOR AND POWER AMPLIFIER

An audio-frequency oscillator can enhance considerably the range of experiments that can be performed on these boards. Most of the commercially available instruments can deliver only a fraction of a watt at a maximum of 20 volts(rms) which is hardly enough for driving a load on these boards. We have, therefore, designed an AF oscillator along with a power amplifier.

The oscillator uses a Wien bridge as a feedback network and a FET as a voltage variable resistance for amplitude stabilization (see the discussion on the Ultra-low frequency oscillator in Appendix IV-A). The frequency of oscillations is decided by the RC combination. The oscillator output is fed to a voltage amplifier of gain 10 through a potential divider (for amplitude control).

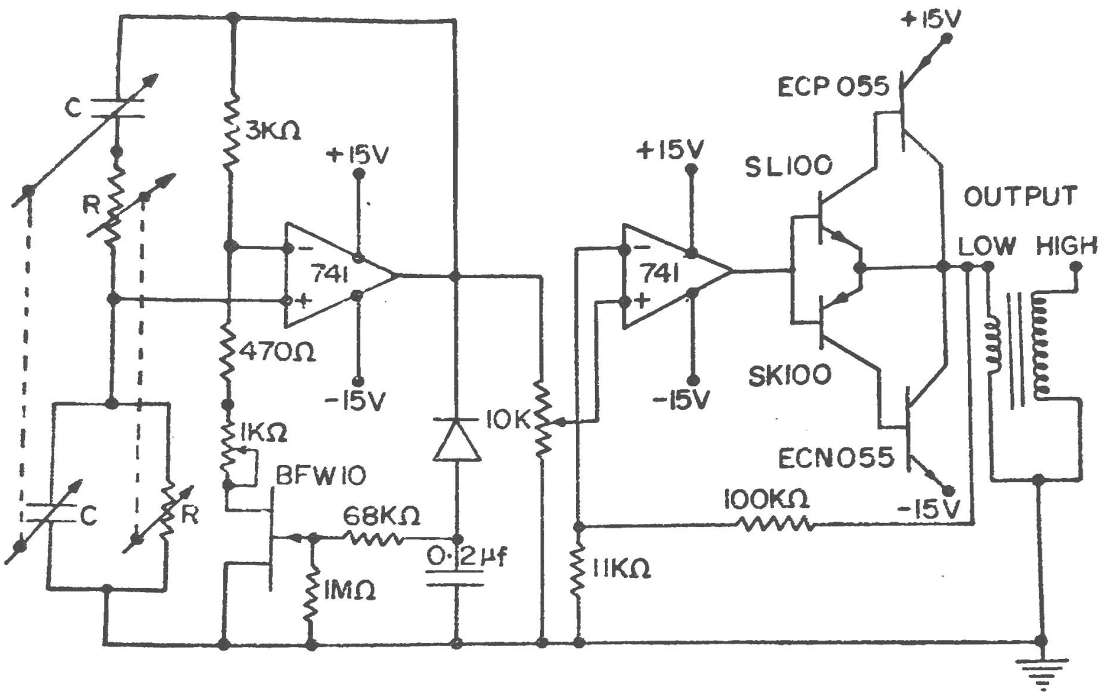


Figure 5.7: FIGURE V-A(1) AF Oscillator with power amplifier

The current amplification is achieved by using a complementary pair of power transistors ECN 055 and ECP 055 which, in turn, is driven by another pair, SL 100 and SK 100. The feedback to the amplifier is taken from the output of the power stage to eliminate cross over distortion. A hi-fi transformer is connected to the output stage to boost it to 100 volts (rms).

The oscillator frequency can be adjusted to any of 10 spot frequencies from 20 hz to 10 khz or can be continuously varied from 30 hz to 3 khz. The power amplifier, which has provision of amplifying external signals also, is capable of delivering 10 w to a 10 ohm load or 7 w to a 1kΩ load. The input sensitivity for rated power is 1 volt rms. The unit is fed with a ± 15 volt, 1 amp regulated power-supply.

VI

# CHAPTER 6

# 6. Study of Inductive Circuits

## 6.1 Introduction

We now extend our study of circuits to include inductors as well. A simple air-core inductor consists of a number of turns of insulated copper (or any other conductor) wire wound on a non-conducting frame of any shape. Often, you will find that strips of some magnetic material such as soft-iron are used for frame. This is done in order to increase the inductance of the system. Use of magnetic cores in a-c circuits, however, results—owing to hysteresis and eddy currents—in unintended but unavoidable *core losses*.

When a current I is set up through an inductor, it stores up an energy $\frac{1}{2}LI^2$ in its magnetic field where L is its inductance (Appendix VI-A) or, to be more precise, its self inductance * (Compare this expression with $\frac{1}{2}CV^2$, the expression for energy stored by a capacitor. In a sense, the energy in an inductor is stored in its magnetic field whereas in a capacitor, it is stored in its electric field).

The *back or induced emf* developed across an inductor while the current in the circuit is changing, is given by (Appendix VI-A)

$$E = -L\frac{dI}{dt} \tag{6.1}$$

where E is measured in volts, L in henrys and $di/dt$, the rate of change of current, in amp/sec. A henry is the inductance that develops a back emf of 1 volt when the current in the circuit changes at the rate of 1 amp/sec.

## 6.2 An LR Circuit with a Source of Constant EMF

Energy may be delivered by a source to an inductor or the stored energy in an inductor may be released in an electrical network and delivered to a load. For example, look at the circuit in Fig. 6.1. An inductor of 2,000 h having a resistance of 200 ohms is connected in parallel with a small resistor of 20 ohms. These are then connected across a d-c supply giving 10 volts. When you turn on the switch S and look at the two ammeters, you will find that the current in the resistive arm will quickly reach its maximum value of about 0.5 amp. whereas the current in the Lr arm will take about 5 seconds to reach its maximum value (50 mA approx).

*If the energy is stored in a system of two coils, wound on the same or different cores, you talk of *mutual inductance*. Any common transformer or your induction coil is a good example of such a system. We shall, however, not discuss such systems.

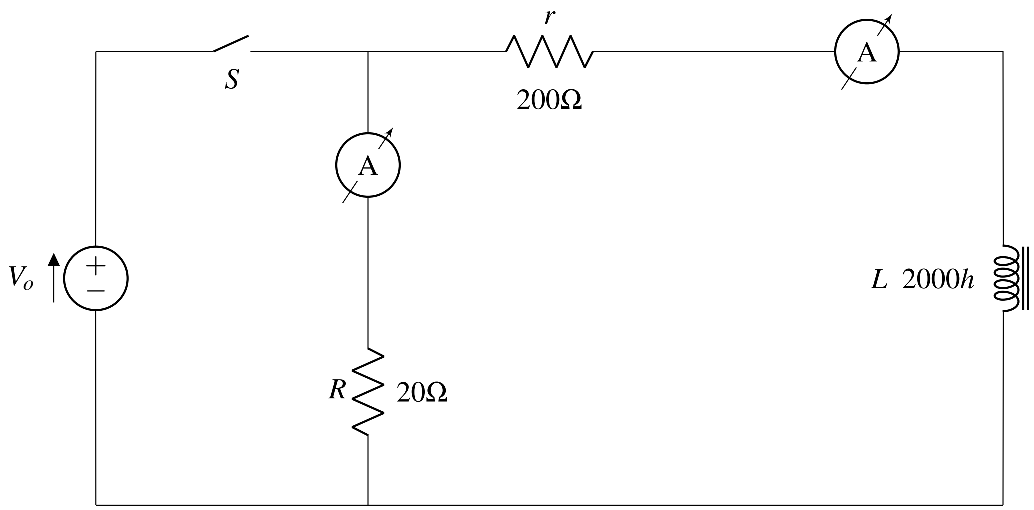


FIGURE 6.1

When you switch off the circuit you will find that the ammeter in the resistive arm not only quickly returns to zero but shows a deflection of 50 mA in the opposite direction and then slowly (in 5 sec. or so) returns to zero. This is because the inductor now delivers the energy $(\frac{1}{2}LI^2)$ stored in it to the load.

It can be shown (Appendix VI-B) that the growth of current in an inductive circuit can be represented by the relation

$$I = I_0[1 - e^{-\frac{R}{L}t}] \tag{6-2}$$

where I is the current at time t and $I_o$ its steady value. Similarly, the decay of current in such a circuit can be represented by

$$I = I_0 e^{-\frac{R}{L}t} \tag{6.3}$$

These expressions clearly tell us that the growth and the decay of current in an inductive circuit depend on the value of L/R and are exponential in nature. Equation (6.3) shows that L/R is the time in which the current drops to $(1/e)$ of the initial value $I_o$. Further, since L/R has dimensions of time, it is called the *time constant* of the circuit.

It is useful to compare Equations (6.2) and (6.3) with equation (2.2) and (2.3) respectively for RC circuit. There the *time constant* was RC.

## 6.3 An LR Circuit with a Source of Alternating EMF

Fig. 6.2(a) shows a 'pure' inductor L and a resistor R connected in series to an a-c source. If the current in the circuit be $I_o\ cos\ \omega t$, a simple analysis (Appendix VI-B) tells us that the emf across the inductor is given by

$$V_L = \omega L I_o\ cos\ (\omega t + \pi/2) \tag{6.4}$$

Equation (6.4) shows that we can associate a reactance of magnitude $\omega L$ with an inductor, this being the ratio of peak voltage $\omega L I_o$ to the peak current $I_o$. It also shows that when a 'pure' inductor L is in series with a resistor R, the voltage across it leads the current I (hence the voltage $V_R$) by $\pi/2$ [see Fig. 6.2(b)].

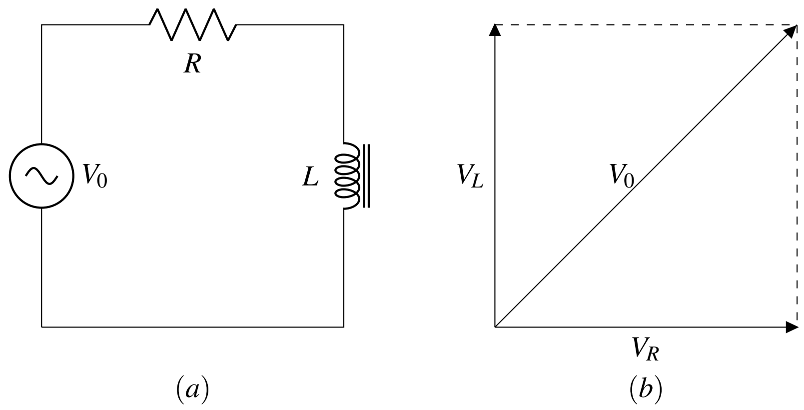


FIGURE 6.2

In reality, every inductor inevitably has some resistance r—called its d-c resistance—associated with it. This comes from the resistance of the winding which would be finite unless one has a superconducting winding! More over, there are also core losses which play an important role in a-c. Effectively, these losses are also represented by a series resistance, sometimes referred to as the a-c resistance. Normally, the series resistance r is deemed to incorporate both the d-c as well as the a-c resistance of the inductor. Thus the equivalent circuit is always like the one shown in Fig. 6.3(a).

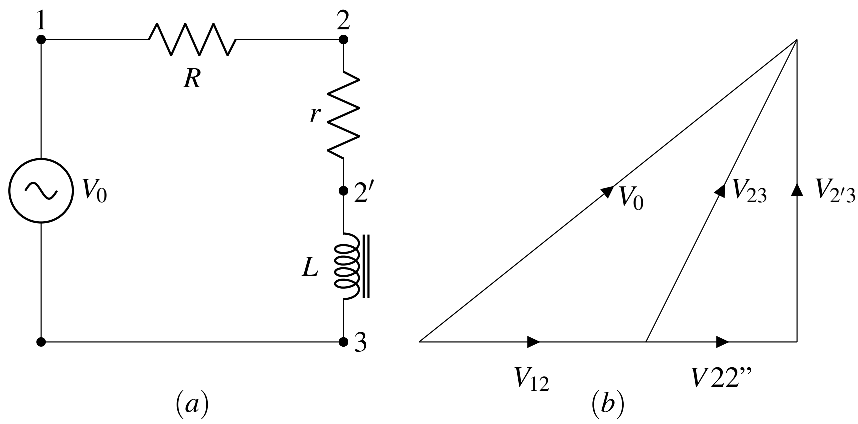


FIGURE 6.3

When we measure $V_L$, it always means the voltage across L and the series resistance r, i.e.

$$V_L =\approx V_{23}$$

This voltage will not, of course, lead the current by exactly $\frac{\pi}{2}$. In fact, the voltages $V_{12}$, $V_{22'}$, $V_{2'3}$ and $V_{23}$ will look like Fig. 6.3(b). The voltage across an actual inductor ($V_{23}$ can be seen to lead $V_{12}$, the voltage across the resistor, by an angle less than $\pi/2$. In this sense, an inductor differs from a capacitor which, for the laboratory voltages used, has a very little leakage and behaves as a 'pure' capacitor. We have already discussed transients (Appendix IV-C) and the relatively unimportant role of the time constant in an RC circuit with a-c. You should be able to discuss the case of an LR circuit with a-c in a similar fashion.

## 6.4 LCR Circuit with d-c Source

For a simple analysis of an LCR circuit with a d-c source, we may refer you to any of the standard texts on the subject.

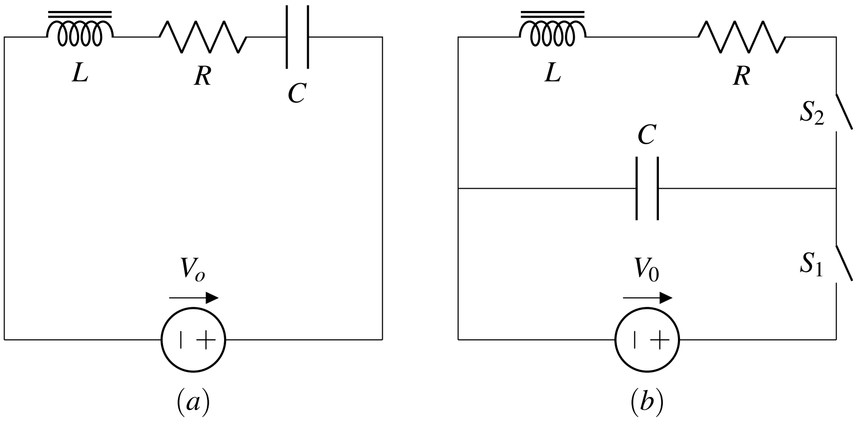


FIGURE 6.4

For the case of charging [Fig. 6.4(a)], such an analysis yields

$$q = -\frac{q_0 e^{-\frac{b}{t}}}{2}\left[\left(1+\frac{b}{m}\right)e^{mt} + \left(1-\frac{b}{m}\right)e^{-mt}\right] + q_0 \tag{6.5}$$

where $m = \sqrt{b^2 - k^2}$; $b^2 = \frac{R^2}{L^2}$ and $k^2 = \frac{1}{LC}$

If $R^2 > \frac{4L}{C}$, Equation (6.5) tells us that the charge on the capacitor will grow to $q_0$ aperiodically. On the other hand, if $R^2 < \frac{4L}{C}$ then Equation (6.5) can be solved to show that the current in the circuit and the charge on the capacitor will grow in an *oscillatory* fashion whose *time period* would be given by

$$T = \frac{2\pi}{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}} \tag{6.6}$$

and the *frequency* by

$$f = \frac{1}{T} = \frac{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}}{2\pi} \tag{6.7}$$

It can be seen from Equation (6.6) that $R^2 = \frac{4L}{C}$ would represent the limiting case when the behaviour of the circuit just ceases to be oscillatory. Similarly, for the *discharging* of the capacitor [Fig. 6.4(b)], the analysis yields

$$q = \frac{q_0 e^{-\frac{b}{t}}}{2}\left[\left(1+\frac{b}{m}\right)e^{mt} + \left(1-\frac{b}{m}\right)e^{-mt}\right] \tag{6.8}$$

The conditions stated in the foregoing case are valid for this case also and determine the mode of discharge of the capacitor. The expressions for the time-period and frequency also remain the same.

## 6.5 LCR Circuit with an a-c Source

It can be seen from Equation (6.8) that even when the circuit is oscillatory, the oscillations are *damped* and will eventually die out. The time for this to happen will depend on $L/R$ and hardly exceeds a few seconds in practice.

If the LCR circuit is driven by an a-c source, very soon the natural oscillations (transients) of the circuit die out and the circuit responds to the applied signal; this means that all the relevant quantities (charge, current, voltage, energy) will oscillate with the *applied* frequency. This, then, is the *steady state* situation and you must always remember that it is this we are usually talking about in the a-c experiments. In fact, the vector diagrams refer only to this steady state.

For a series LCR circuit of Fig 6.5 our voltage vector diagram looks like

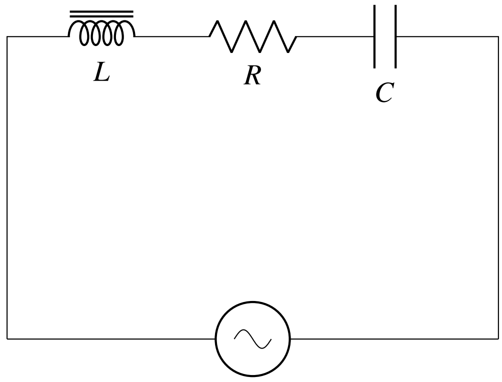


FIGURE 6.5

that in Fig. 6.6(a) (For simplicity we assume that R includes the resistance r of the inductor coil as well). Here OA, OB and OC are the voltages across R, L and C respectively.

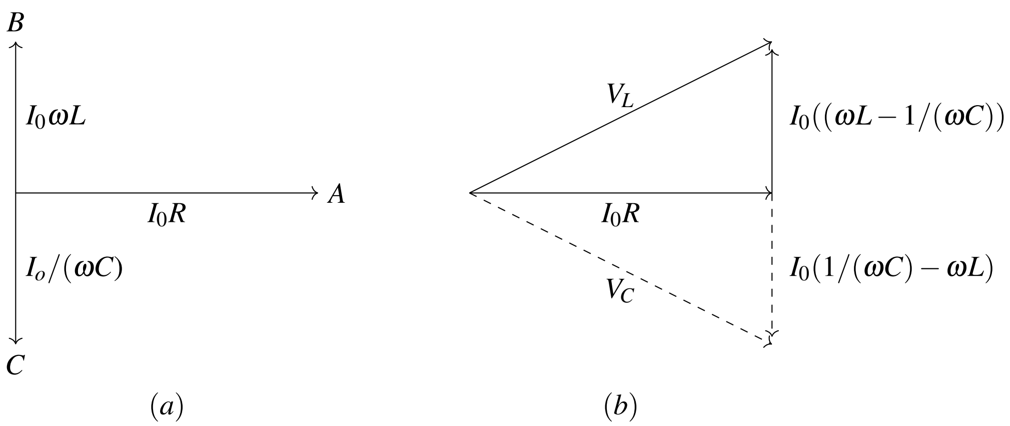


FIGURE 6.6

If ($I_o$ is the peak current, all the voltages also will be the peak voltages. Alternatively, the current and the voltages can all be rms values). If $\omega L > \frac{1}{\omega C}$ the net voltage across L and C is clearly $I_o\left(\omega L - \frac{1}{\omega C}\right)$ along OB so that the sum of all the three vectors has the magnitude

$$I_o\sqrt{R^2+(\omega L-\frac{1}{\omega C})^2}$$

The angle $\theta$ by which the applied voltage *leads* the current is given by

$$tan\ \theta = \frac{(\omega L - \frac{1}{\omega C})}{R}$$

If, on the other hand, $\omega L < \frac{1}{\omega C}$, the applied voltage will lag behind the current and all the expressions would still be true; only $\theta$ will be negative now.

If $\omega L = \frac{1}{\omega C}$ then $\theta = 0$. The emf and the current in this case would be in *phase* and the impedance of the circuit would be purely resistive in nature. This is the case of *resonance* and the *resonant frequency* is given by

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

## 6.6 The Network Board - 6

The lay out of this board is shown in Fig. 6.7. In addition to a number of inductors, resistors and polystyrene capacitors, a transformer with various tappings for 10 to 100 volt and capable of handling a maximum current of 100 mA is also provided. A 50 mA d-c meter along with a bridge rectifier and suitable series resistors serves to measure a-c voltages. The board is, of course, suitable for studying inductive circuits with a-c only. We shall now describe some experiments with such circuits for both a-c and d-c. For performing experiments with d-c, you are advised to procure suitable inductors and other components (described in the relevant experiments) and assemble the network yourself.

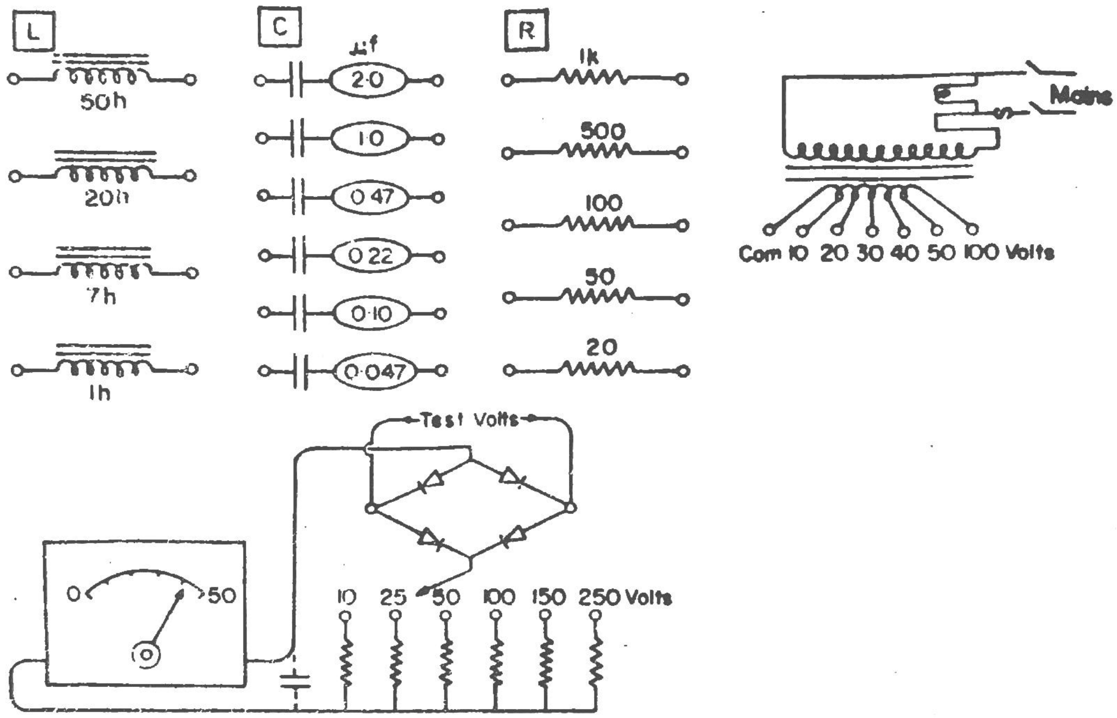


FIGURE 6.7

## 6.7 Experiments

### 6.7.1 Growth and decay of current in LR circuit

#### Experiment VI-A: To study the growth and decay of current in an LR circuit.

(i) **Inductor with Magnetic Core**

The experiment is similar to that for the study of charging and discharging of a capacitor (Experiments 2-A and 2-B). For this experiment you require an inductor of large L/R (10 sec or more) so that you may be able to make the measurements directly with the help of a current meter and a metronome as the current grows or decays. To make L/R large one may decrease R (using thick wire) or increase L (using many turns in the coil). If one makes the wire too thick one cannot increase the number of turns keeping the overall size of the coil reasonably small. Therefore, we pay attention to another method of increasing L, viz., the use of an iron core. The inductor supplied with the network board has L/R value of 10 sec or so. It may be noted that iron core inductors have the problem that their inductance L varies considerably with the current and the behaviour during rise of current is different from that during fall (hysteresis). But if large L/R is needed we have no alternative (unless superconductor wires are used)!

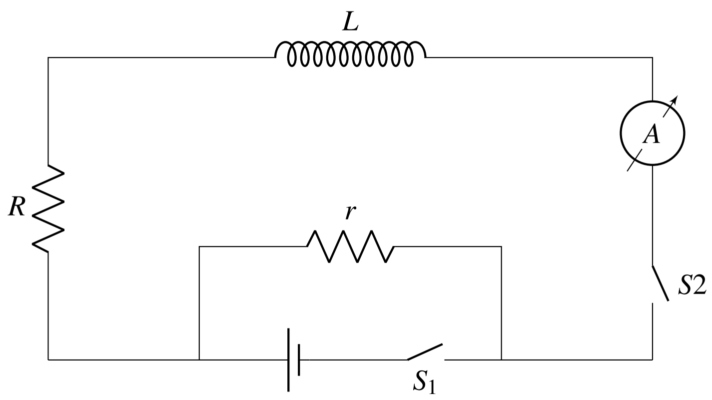


FIGURE 6.8

The circuit is shown in Fig. 6.8, where L represents the pure inductance and R represents pure resistance, including those of the inductor, the meter and the actual resistor in the circuit.

To study the growth of current you first close the switch $S_1$ then close the switch $S_2$ and start the metronome simultaneously. You note the current in the meter at regular 'ticks' of the metronome, until the current reaches a steady value. Now you stop the metronome.

After some time you open the switch $S_1$ and start the metronome simultaneously. You note the current in the meter at regular 'ticks' of the metronome, until the current falls practically to zero. Note that the battery is shunted with a resistance $r \ll R$ to ensure that circuit resistance during decay remains close to R; in the absence of this shunt it would become infinite and sparking would occur when $S_1$ is turned off. In fact r could not be too small either, since that would short-circuit the battery; we have kept r≈20 ohm.

Figure 6.9 represents the results for both growth and decay of current.

However, even with L/R ratio ≈10 sec., it is not very convenient to make the measurements of I and t directly.

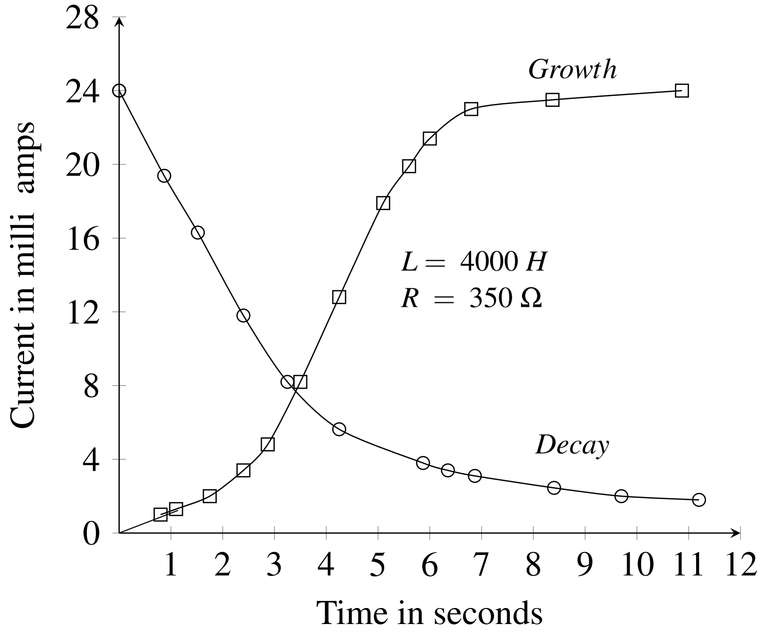


FIGURE 6.9 Growth and decay of current in iron core inductor (Data using a metronome)

;

So we have repeated this experiment with the help of a 'timer'. (The use of this is discussed below under the heading 'Air core inductor'). The I, t graphs obtained with the timer are given in Fig. 6.10(a).

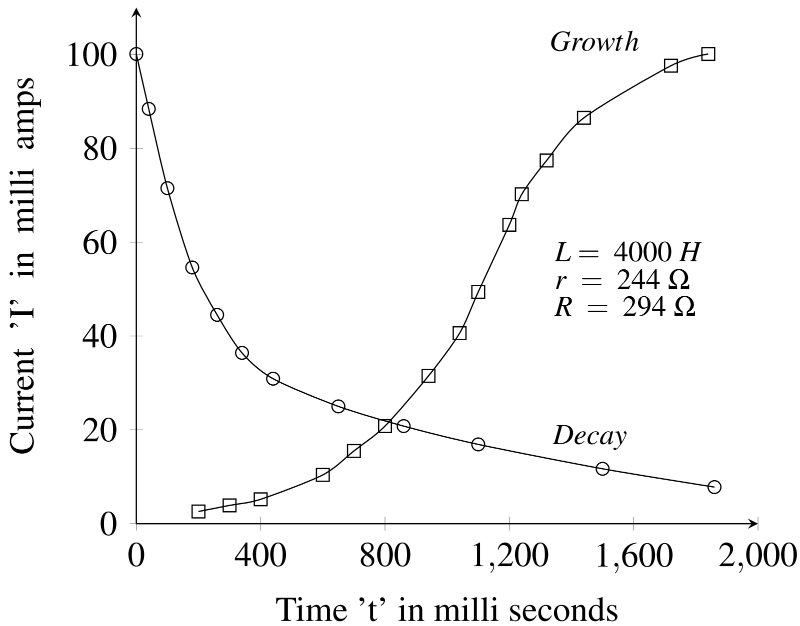


FIGURE 6.10(a) Growth and decay of current in iron core inductor (Data taken by 'timer')

To understand these graphs, we write Equations (6.2) and (6.3) in the following form—

$$log(I_0 - I) = logI_0 - \frac{R}{L}t \tag{6.9(a)}$$

and

$$logI = logI_0 - \frac{R}{L}t \qquad (6.9(b))$$

Thus if we plot $I_0$¯$I$ or I, as the case may be, on a *log scale* against time, we ought to get a straight line if the growth and decay are exponential.

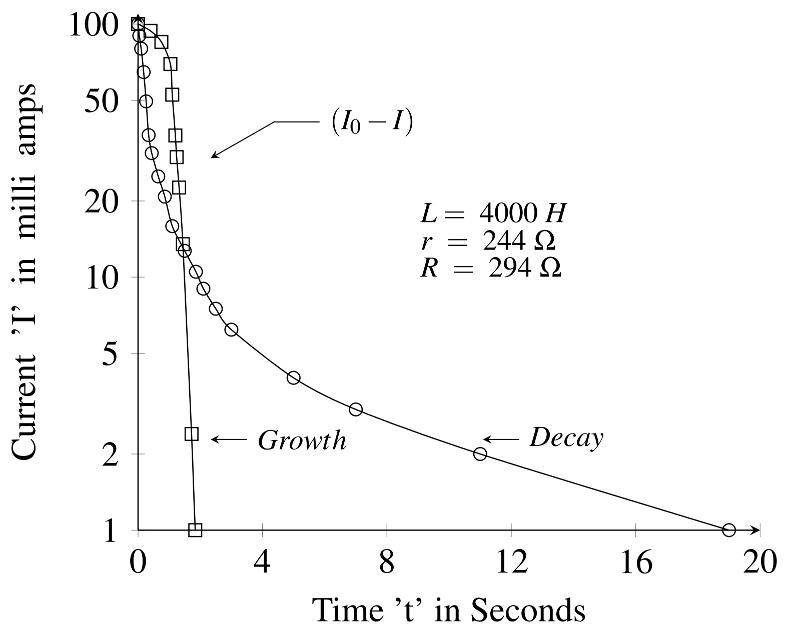


FIGURE 6.10(b) Growth and decay of current in iron core inductor (Semilog plot of data using a 'timer')

Fig. 6.10(b) shows such plots—and it is evident that there are serious deviations from the expected behaviour. In the theory we have treated L as a constant of the inductor; but in iron-core inductors this is not true. We would not go here into the details, except to record this departure, which reflects the behaviour of the core material under magnetization.

**(ii) Inductor with-Air Core**

For an air-core inductor L/R can be raised at best to a few millisecond. For measurements we have to use an electronic device called the *digital timer*. (For description see 'Physics Through Experiment' Vol. 2). This timer can be set into several modes, two of which we will use. It measures time interval between two pre-specified states with least count (typically) 0.1 milli-sec.

To study the growth of current make connections as shown in Fig. 6.11(a). The timer is set in A = B mode, which means that the timer starts with a voltage pulse reaching the A input of the timer (see left top), and stops when another pulse reaches the B input value $V_r$. (Fig. 6.11 (a)]. By repeating the experiment a number of times, setting $V_r$ at a different value each time, you get the required data for the growth of current with time.

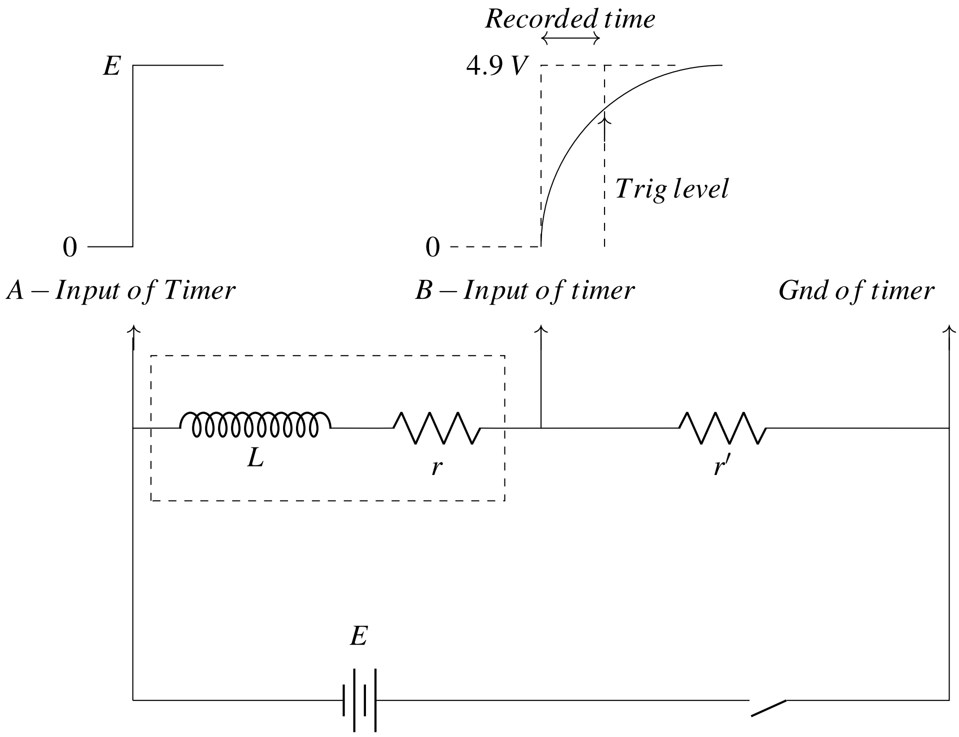


FIGURE 6.11(a)

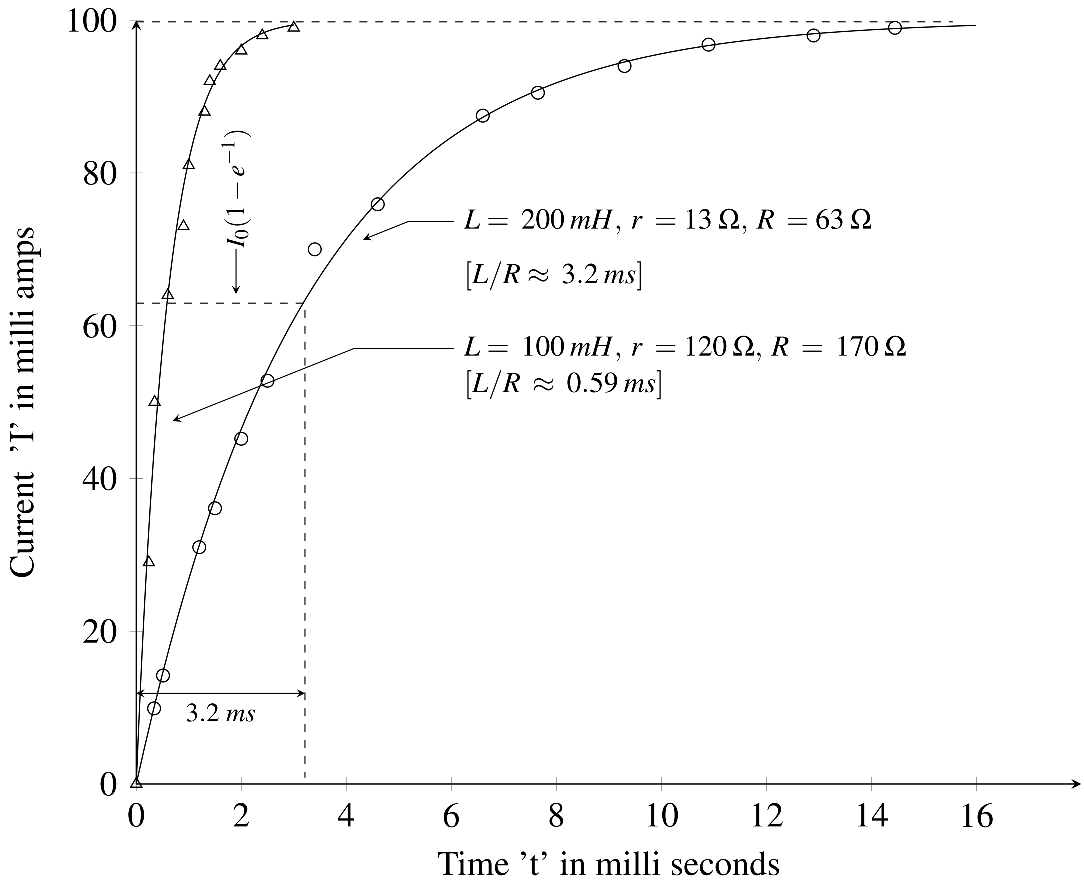


FIGURE 6.11(b)

The pulses have to be above the *trigger* level. You set the trigger level of A input at zero and of B input at various predetermined values, say, $V_r$. Thus the timer really measures the interval from the switching time to the time when the current reaches the value $V_r$ [Fig. 6.11 (a)]. By repeating the experiment a number of times, setting $V_r$ at a different value each time, you get the required data for the growth of current with time.

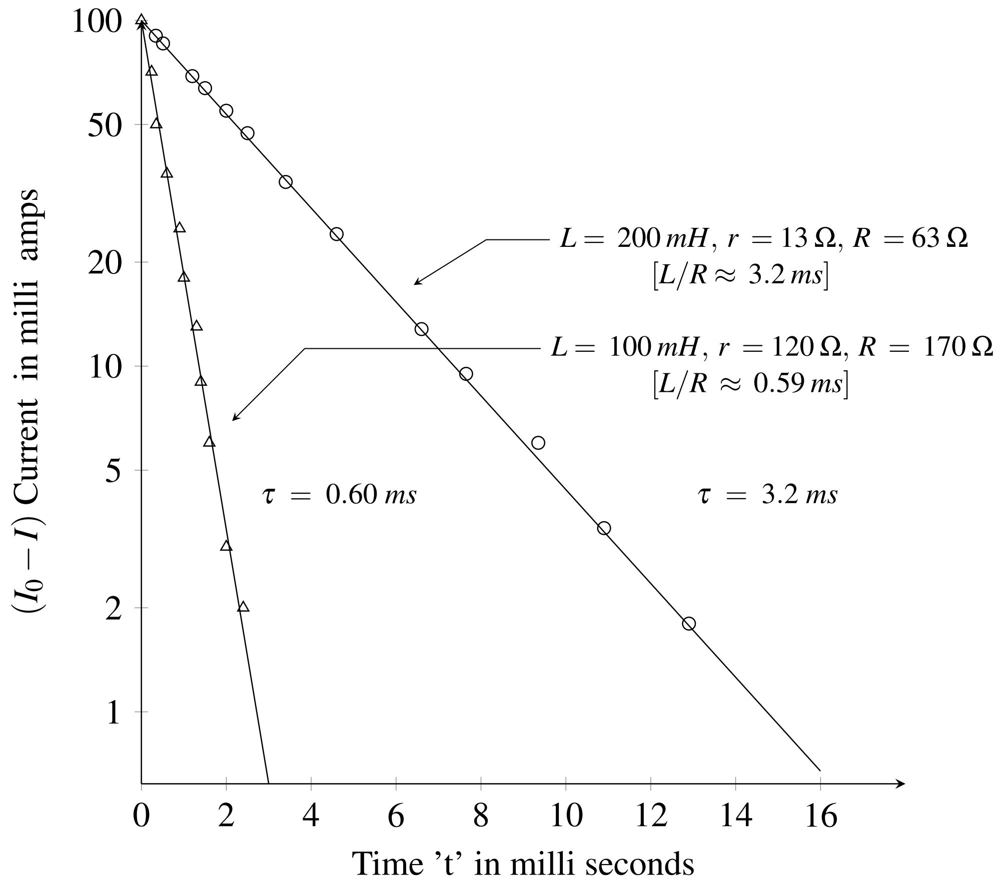


FIGURE 6.11(c)

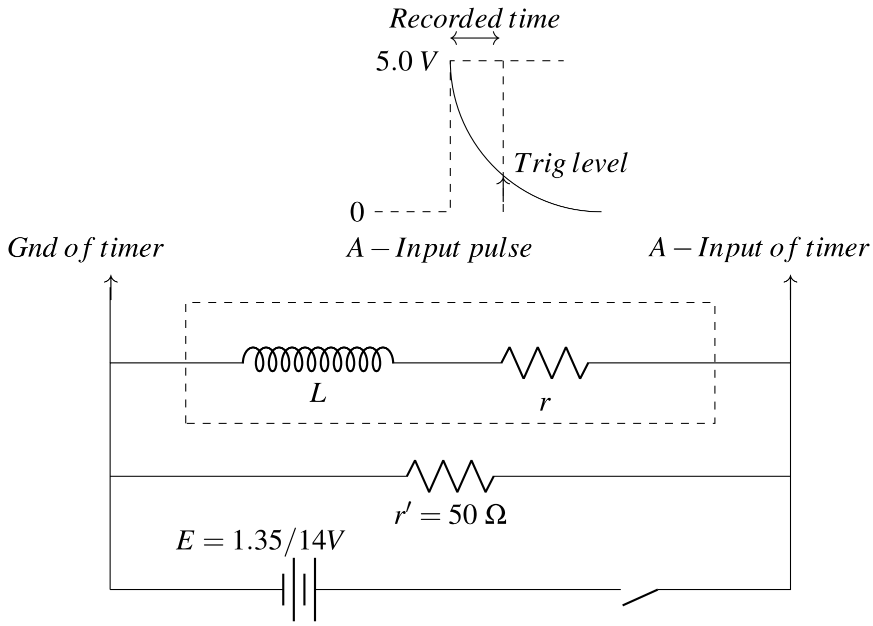


FIGURE 6.12

Figures 6.11(b) and 6.11(c) show the experimental curves for I vs.t and $log(I_0{}^{-}I)$ vs. t respectively. Note the fact that the semilog plots come out as straight lines as expected from theory and that the slopes agree closely with the ( L/R ) values.

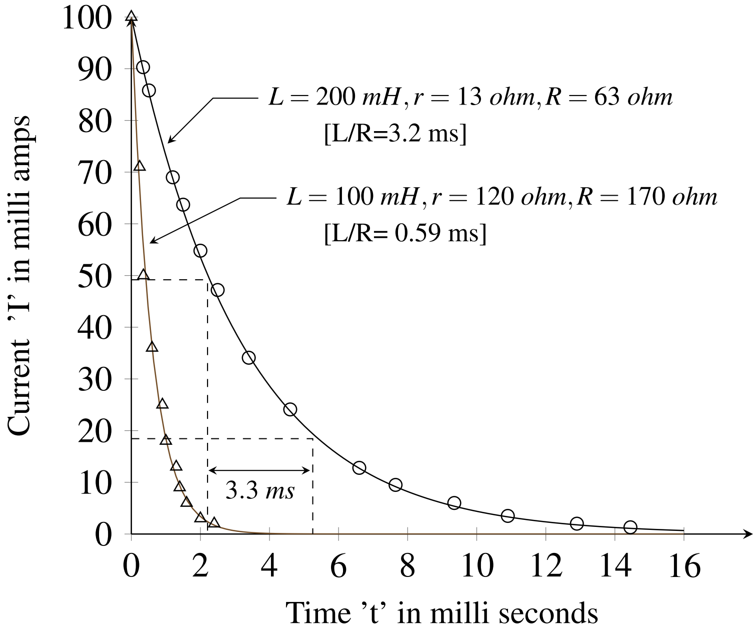


FIGURE 6.13(a)

To study the decay of current make connections shown in Fig. 6.12, using the timer in mode A. In this mode the timer starts when a pulse of height greater than its trigger level reaches the A input and stops when the pulse height reduces to a value equal to its trigger level. By setting different values of the trigger level you may obtain the different times in which the current falls from $I_o$ to different I values. Figures 6.13(a) and6.13(b) show the typical experimental results.

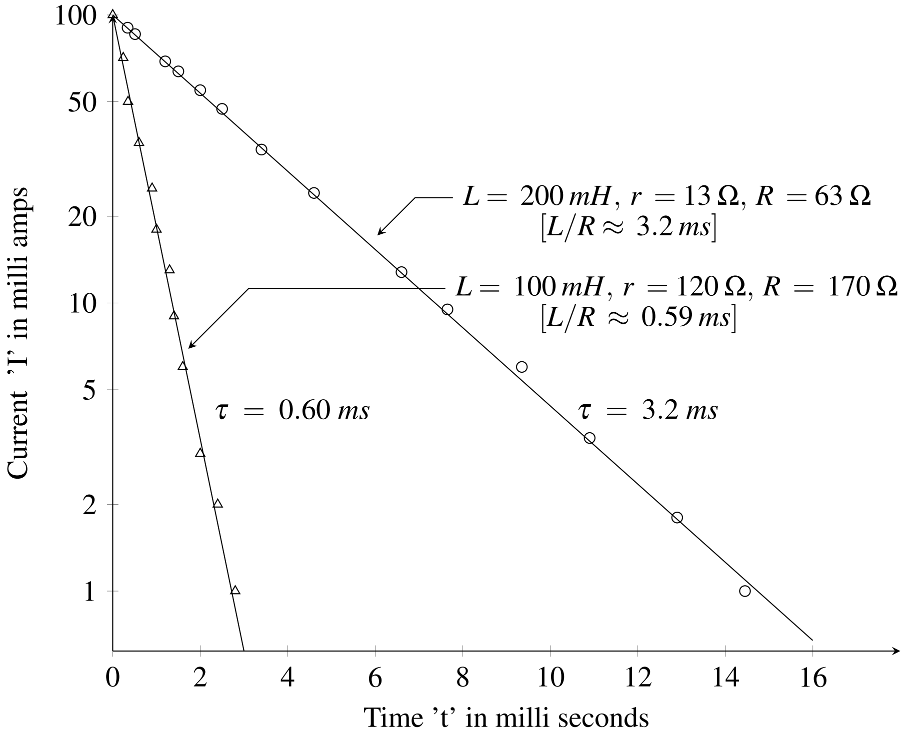


FIGURE 6.13(b)

## 6.7.2 Energy stored in inductor in LR circuit

### Experiment VI-B: To Measure the Energy Stored in an inductor in LR circuit.

We have said that the energy stored in an inductor, on setting up a steady current $I_o$, is $\frac{1}{2}LI_0^2$. Determine this in the same fashion as you did in the case of a capacitor (Experiment 2-E). You can do this both when the current in the circuit is growing and when it is decaying. The area under the $I^2$-t curves would be found to be proportional to the stored energy. Measure the instantaneous values of the current as in the preceding experiment.

## 6.7.3 Charge-discharge of a capacitor in LCR circuit

### Experiment VI-C: To study the charging and discharging of a capacitor in an LCR circuit.

The circuit for studying the charging is shown in Fig. 6.4(a) and that for the discharging in Fig. 6.4(b). For the oscillatory charge and discharge of the capacitor $R^2$ should be less than $4L/C$. Further, in order to make the period of oscillation large, both L and C should be as large as possible. See Equation (6.6). Choose a small value of R such that $R^2$ becomes much less than $4L/C$. For studying the other case, make $R^2 > 4L/C$ by choosing a large value for R. Plotting $V_C$ as a function of time would yield the curves shown in Fig. 6.14(a) for the charging and those in Fig. 6.14(b) for the discharging of the capacitor.

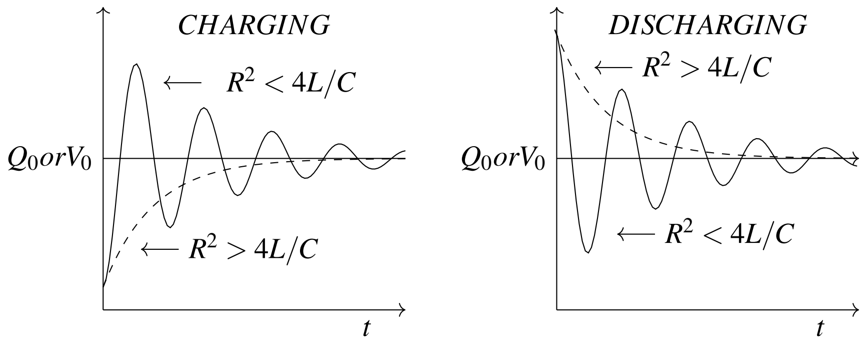


FIGURE 6.14

For obtaining good results, use an inductor with a large $L/R$ value and a capacitor having a large capacitance (a few electrolytic capacitors of 2000 $\mu$f or so, joined in parallel, should do the trick). Remember, in the oscillatory charging, $V_C$ may shoot up to a value as high as twice the supply voltage and hence the latter must be kept well below half the breakdown voltage for the capacitors. These capacitors cannot be used as such in studying the oscillatory discharge, for the polarity across these keeps on reversing [Fig. 6.14(b)].

## 6.7.4 Equivalent power loss resistance of inductor

### Experiment VI-D: To determine the equivalent power loss resistance of an inductor.

Connect the circuit of Fig. 6.15, where r is the resistive part of the inductor and is to be determined. Measure $V_L$, $V_r$ and $V_0$ (the applied voltage) and draw the voltage triangle. You will find that the triangle is not a right angled one. Draw a semi-circle with $V_o$ as diameter and extrapolate the $V_r$ line until it intercepts the semi-circle. You can now estimate the equivalent power loss resistance (from $V_r$) of the inductor since $r/R = V_r/V_R$.

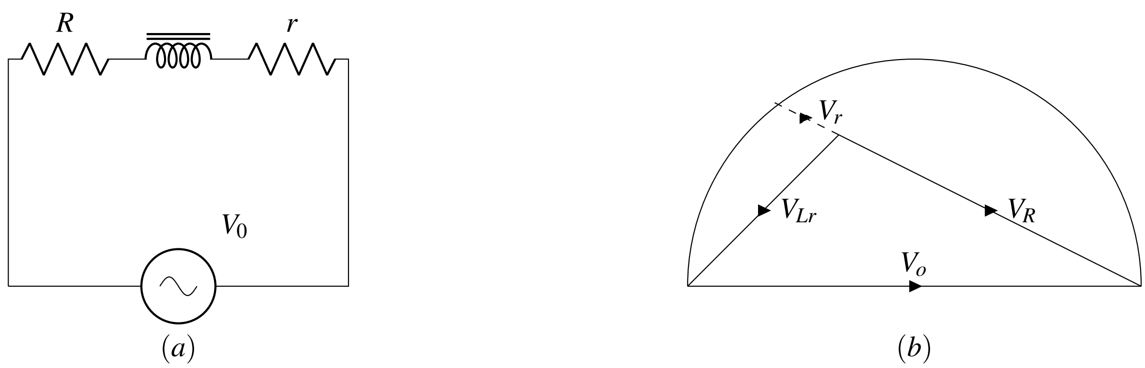


FIGURE 6.15

Compare it with the d-c resistance of the coil (measure it directly with a meter). You will find that there is a difference—the equivalent power loss resistance r is not just the d-c resistance. This is because the power loss in an inductor is due to its d-c resistance plus the hysteresis and eddy losses in the core. Measure this equivalent resistance for different values of the a-c voltage. Why may it vary?

## 6.7.5 Vector diagrams for complex LR circuit

### Experiment VI-E: To analyze a complex LR circuit by drawing vector diagrams.

Make two parallel branches of LR and connect them across a source $V_o$ .

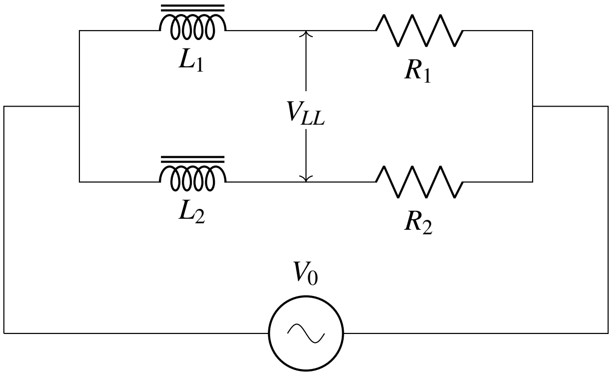


FIGURE 6.16

Measure $V_{L_1}$, $V_{R_1}$, $V_{L_2}$ and $V_{R_2}$ and draw the voltage triangles for the two branches. ($V_{L_1}$, $V_{R_1}$ and $V_0$ for one and a similar one for the other). Now measure $V_{LL}$. See if you can also determine it from the triangles and compare. There are five different voltages in the circuit—determine the phase difference between any two of them from your vector diagrams.

## 6.7.6 Vector diagrams for complex RC circuit

### Experiment VI-F: To analyze a complex LR circuit.

Make two parallel branches of RC (exactly as in the last experiment but replacing L by C) and repeat the measurements indicated in Experiment 6-E. Compare the results of these two experiments.

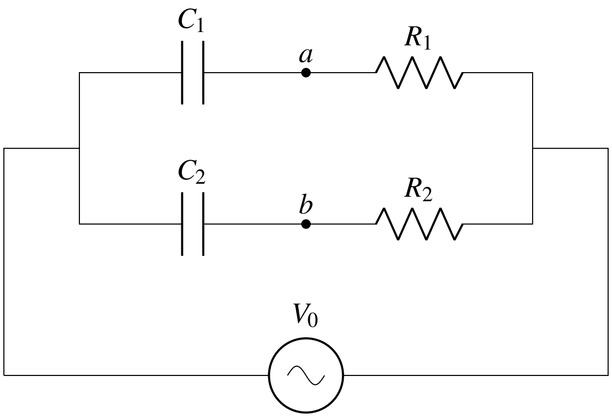


FIGURE 6.17

Assume that $R_1$=$R_2$ and $C_1 = C_2$. Show that the potential difference between a and b (Fig. 6.17) is independent of the applied frequency. Suppose the source had a small resistance (say $R_1/10$). What effect will it have on the magnitude of $V_a - V_b$ and its phase relative to $V_0$ ?

## 6.7.7 Inductors in series

### Experiment VI-G: To study a circuit with two inductors in series.

Connect two inductors and a resistor in series in any order. Measure the voltages across the inductors, the resistor and the source. Construct the voltage vector diagram as a polygon (Fig. 6.18) and determine all the phase angles.

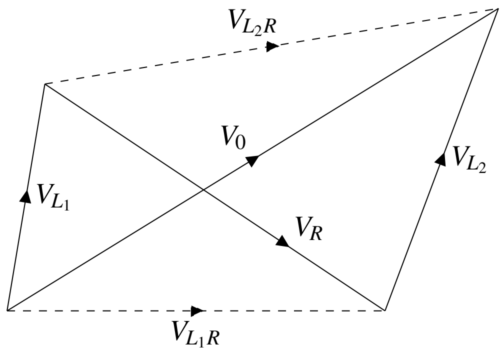


FIGURE 6.18

Rearrange the order of $L_1$, R and $L_2$ and construct the relevant polygon. Of course, the resultant $V_o$ is the same, only the vectors change their sequence.

## 6.7.8 Capacitors in series

### Experiment VI-H: To study a circuit with two capacitors in series.

The Experiment 6-G may be repeated with capacitors in place of inductors. Again draw all the relevant polygons and determine the phase angles.

## 6.7.9 Phase difference between $V_C$ and $V_L$

### Experiment VI-I: To study if $V_L$ and $V_C$ are always in the opposite phase.

Make a circuit as shown in Fig. 6.19 (a) and measure all the voltages. Construct voltage triangles for each branch. Can you see why we must draw the triangles on either side of $V_o$ and not on the same side? Which is the vector representing $V_{LC}$? Check its magnitude by measurement. Determine the phase difference between $V_L$ and $V_C$. You will find it is not $\pi$. Try to give a qualitative explanation for this.

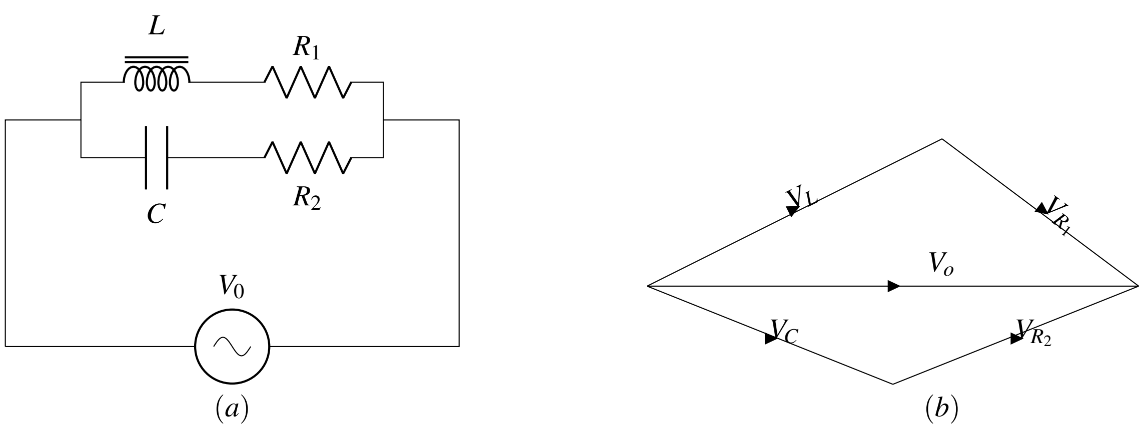


FIGURE 6.19

## 6.7.10 Impedance in LCR circuit

### Experiment VI-J: To study the impedance of an LCR circuit.

Connect an inductor and a capacitor in series to a source. Measure the voltages and construct the voltage triangle.

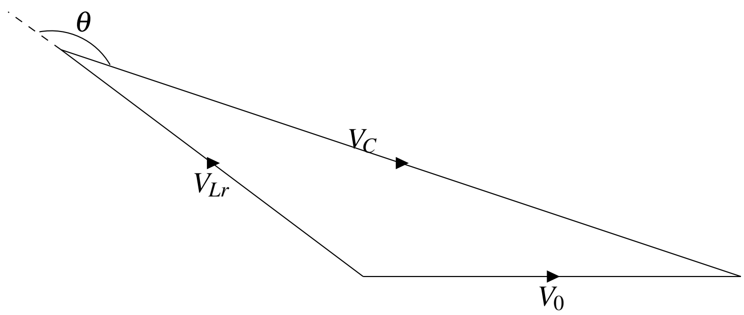


FIGURE 6.20

But for the fact that the capacitor may be leaky and an inductor is never loss less, $\theta$, the phase difference between $V_L$ and $V_C$ would have been $\pi$. Measure the current in the circuit by connecting (in series) a small resistor and measuring the voltage across it. You can now determine the impedance from $Z = \frac{V}{I}$. See if you can establish the relation

$$Z^2 = r^2 + (\omega L - \frac{1}{\omega C})^2 \tag{6.10}$$

where r is the effective resistance associated with L as in Experiment 6-D. (Strictly speaking, r includes all the other effective resistances in series such as the one you have used for measuring the current).

## 6.7.11 Resonance in LCR circuit

### Experiment VI-K: To study the phase relationship in a series LCR circuit.

Connect an inductor and a capacitor in series to the source. Measure the current in the circuit, keeping the source voltage and the inductance the same and trying different values for C by adding capacitors in parallel. Measure the voltages $V_o$, $V_C$ and $V_{Lr}$ and draw voltage triangles using $V_o$ as the common base for all triangles. You will find that the vertices will lie on a circle [Fig. 2.21 (b)]. The current in such a circuit is given by

$$I_0 = \frac{V_o}{|Z|} = \frac{V_o}{\sqrt{r^2 + (\omega L - \frac{1}{\omega C})^2}} \tag{6.11}$$

which has a maximum when $\omega L = \frac{1}{\omega C}$. See if you can verify this. The arm representing the maximum $V_{Lr}$ [see Fig (6.21(a)] is the diameter of the circle referred to. (why ?)

Also construct the dotted lines [Fig. 6.21(a)] representing the voltages across the pure L part and pure r part of the inductor.

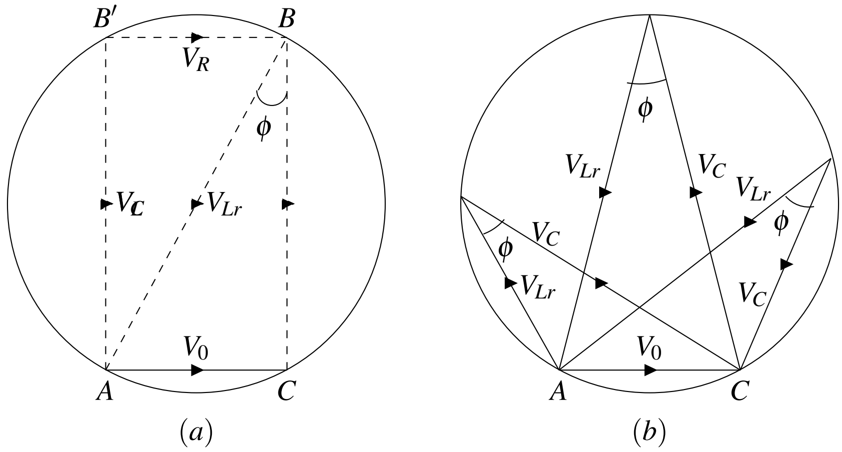


FIGURE 6.21 (a) $V_L$ and $V_C$ can be quite large at resonance compared to $V_o$ (b) For a given $V_o$ and L, the angle between $V_{Lr}$ and $V_C$ is independent of C

This particular situation (maximum current, $\omega L = \frac{1}{\omega C}$ is the case of series resonance. We have

$$\omega^2 = \frac{1}{LC} \tag{6.12}$$

or

$$T = 2\pi\sqrt{LC}$$

where T is the time period of the circuit which, in the case of resonance, would be the same as that of the applied a-c.

Figure 6.21(a) shows that for the case of resonance or near resonance, the applied voltage can be quite small compared to the voltages measured across the inductor and capacitor. This is because $V_{Lr}$ and $V_C$ are nearly out of phase and almost cancel each other. Moreover, the smaller the value of r, the more nearly the two would be out of phase (i.e. the vectors $V_{Lr}$ and $V_C$ will be closer to being antiparallel, the phase difference approaching $\pi$) and the voltages can be very large compared to the applied voltage.

Another important result you ought to note is that the voltage $V_r$ is in phase with the applied voltage. Thus at resonance, the current and the applied voltage are in phase.

You are advised not to make use of the 100 volt tapping near resonance as this may draw a current as high as 100 mA and consequently damage the transformer. Moreover, near resonance even a supply of 50 volts can establish very high voltages across the capacitors and thus damage them. As a precaution, therefore, we should make use of these tappings only when we are working away from resonance. Having drawn the voltage triangle and the circle [Fig. 6.21(a)] for the case of resonance, now draw triangles for other cases on the same base $V_o$.

You will find that all these vertices will also lie on the same circle [Fig. 6.21(b)]; the angle subtended by the chord $V_o$ at the circle is independent of C and is given by

$$tan\ \phi = \frac{r}{\omega L} \tag{6.13}$$

where r is the effective series resistance of L (Of course, for an ideal inductor $r = 0$, and $\phi = 0$). Verify this from your results. From the vector diagram and simple trigonometry you ought to be able to derive this relation.

The angle $theta$ given by

$$tan\ \theta = \frac{\omega L - \frac{1}{\omega C}}{r} \tag{6.14}$$

is termed the phase angle of the impedance. Identify this angle on your vector diagrams.

## 6.7.12 Q of series resonant LCR circuit

### Experiment VI-L: To study the Q of a series LCR resonant circuit.

By now you should have sorted out the appropriate values of L and C that give you resonance (i.e. make $\omega L = \frac{1}{\omega C}$. Now add a small resistance R in series and observe the effect. Try various values of the external resistance R and plot the resonant current (or $V_r$) as a function of $V_R$?. Interpret the results.

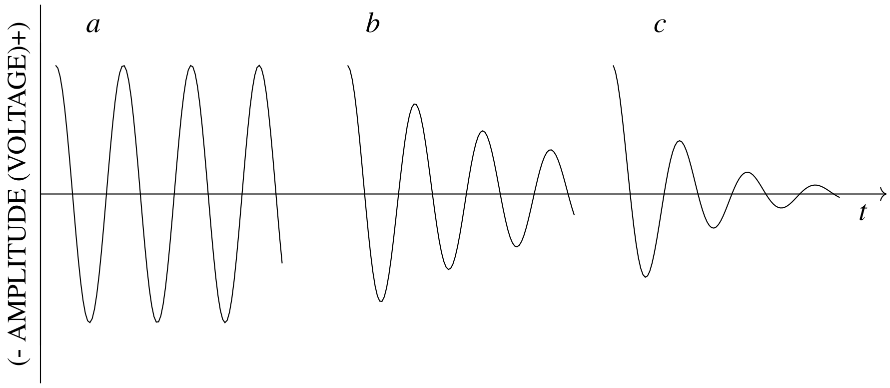


FIGURE 6.22 Oscillations (a) undamped (b) damped (c) heavily damped

The resistive part of an LCR circuit keeps on consuming energy and hence natural oscillations of such a circuit (not fed with an external a-c source) will be damped and progressively die out. A parameter that measures damping in any oscillatory system is the Q of the system. Let us first discuss the meaning of the Q value. Figure 6.22 shows three oscillatory curves. In the curve (a), the amplitude does not change with time. It represents an undamped system. In curves b and c, the amplitude falls off with time. These represent damped systems. It may also be seen from these that the system c is more heavily damped. The less the damping, the larger the number Q. For an oscillator with frequency $\omega$ 'Q' is the dimensionless ratio defined as follows:

$$Q = \omega \frac{energy\ stored}{average\ power\ dissipated} \tag{6.16}$$

or, Q may be defined as $2\pi$ times the number of cycles required for the energy in the oscillator to diminish by the factor $1/e$.

In an LCR circuit the energy is stored alternately in the capacitor and the inductor and the oscillation involves a transfer of this energy back and forth from one to the other. During the course of this transfer, the circuit resistance R keeps on dissipating energy, and as the oscillation goes on, the energy in the system gradually diminishes. For a complete discussion of this you should refer to Berkeley Physics, Vol. II, Chapter 8. It can be shown that Q of an LCR circuit is given by

$$Q = \frac{\omega L}{R} = \frac{\omega L I_o}{R I_o} = \frac{V_L}{V_R} \tag{6.17}$$

where $\omega = \frac{1}{\sqrt{LC}}$ and $I_o$ the resonant current. Thus we see that at resonance $V_L$=$V_C$=$QV_o$.

Generally speaking, the higher the Q of a circuit, the narrower and higher the peak of its response as a function of the driving frequency $\omega$. Further, it can be shown (above reference) that

$$\frac{2\Delta\omega}{\omega_o} = \frac{1}{Q} \tag{6.18}$$

where $2\Delta\omega$ is called the width of the resonance peak (measured at the 'half-power' points) and $\omega_o$ the resonant frequency.

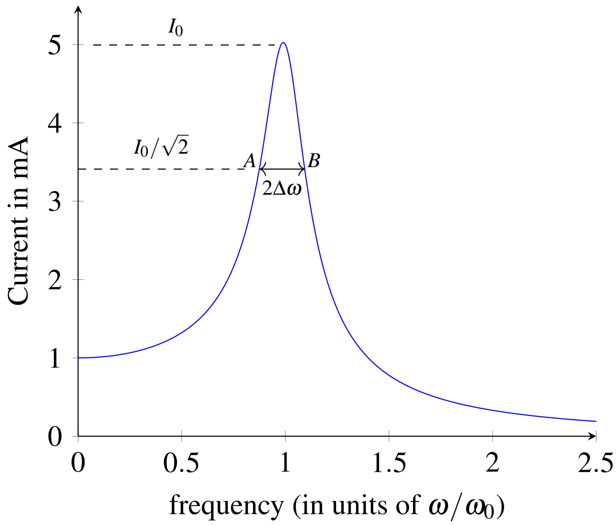


FIGURE 6.23. The width of a resonance peak is expressed by the full width between half-power points

Figure 6.23 is a plot of the response (current) in case of an LCR circuit as a function of the frequency of the applied emf. It can be seen that the current and, hence, the power dissipation in the circuit is maximum at the point of resonance (C). Referred to this curve, $2\Delta\omega$ is the full width of the resonance peak at A, B where the power dissipation drops to half its peak value (or, where the current drops to $\frac{1}{\sqrt{2}}$ of its peak value).

In order to measure the Q of the circuit directly, use an AF oscillator (Appendix V-A) to supply the driving signal and measure the current (or $V_r$) in the circuit as a function of the applied frequency. Plot the graph as in Fig. 6.23, locate the half-power points and determine the Q. Now change the resistance R and plot another graph. Interpret your results.

If you do not have an AF oscillator you can still determine the Q of the circuit approximately by keeping the applied frequency fixed at 50 hz and studying the variation of the current (or $V_R$) if you go on changing C.

You plot the current $I$ as a function of $\frac{1}{\sqrt{c}}$ you will obtain a curve that resembles the resonance curve (the curves in Fig. 6.24 have been so plotted) and you may read off the Q as before. But you must remember that in changing the capacitor you are really changing the system itself and what you get is really only an order of magnitude estimate of the Q of the circuits that you use. When you estimate the Q for the curves in Fig. 6.24, you will find that the lower curve has a higher Q. In the context of the statement made earlier the higher the Q of a circuit, the narrower and higher the peak of its response—do you find this result paradoxical?

You will find that the Q of these systems is about 10 or so. However, systems with very much higher Q than this are quite common. Your radio receiver has resonant circuits of the order of several hundred Q whereas a micro wave cavity may have a Q of $10^5$. A laser has a very high Q, of the order of $10^{11}$.

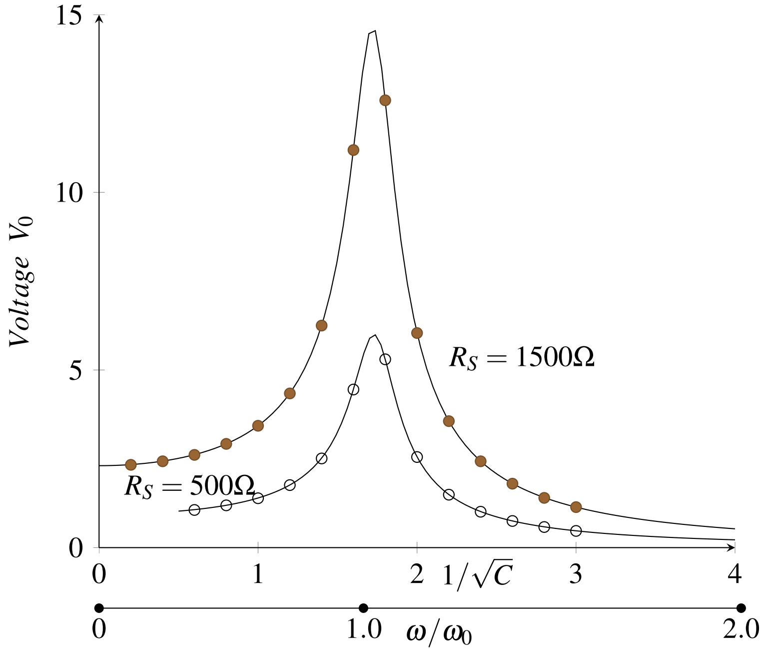


FIGURE 6.24. Resonance curves plotted at the frequency of the mains

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# 6.8 APPENDIX

## 6.8.1 Appendix VI-A INDUCTANCE

When a steady current flows through a coil it produces an attendant magnetic flux. If the current is now changed, this flux also changes. According to Faraday's and Lenz's laws (discussed in Appendices VIII-A and B), this produces an emf which opposes the change of current in the coil.

Since the magnetic flux is proportional to the current in the conductor, the rate of change of flux is proportional to the rate of change of current. Thus the induced emf is

$$e = -\frac{d\phi}{dt} = -L\frac{di}{dt} \tag{VI-A-1}$$

where L is positive constant for the coil and is called the self-inductance. $L$ is a measure of the ability of the coil to oppose a change of current through it. The negative sign is important. It indicates that the induced emf has a sign opposite to that of $\frac{di}{dt}$. If $\frac{di}{dt}$ is positive i.e. when when the current in the circuit is increasing, e acts in a direction opposite to that of the current. On the other hand, when the current is decreasing, the induced emf acts in the direction of the current.

Thus, in order to increase the current in a circuit the applied emf has to do work against this back emf. The rate at which it will have to do work at any instant is given by $LI\frac{dI}{dt}$. The work done during a small interval of time dt would be LI dI. The total work done in order to set up a steady current $I_o$ in the circuit, starting from zero current, can be calculated as follows:

$$W = \int dW = \int_0^{I_o} LI\,dI = \frac{1}{2}LI_o^2 \tag{VI-A-2}$$

This energy is stored up by the inductor in its magnetic field. Note that this energy stored is independent of the time spent in setting up the current $I_o$ -a result akin to the one obtained for the energy $\frac{1}{2}CV_0^2$ stored by a capacitor.

## 6.8.2 Appendix VI-B ANALYSIS OF LR CIRCUITS

### 6.8.2.1 AN LR CIRCUIT WITH A D-C SUPPLY

The emf equation for an LR circuit with a constant source of emf is written as

$$L\frac{di}{dt} + IR = V \tag{VI-B-1}$$

This means that the applied emf divides in two parts, $LdI/dt$ across the inductor and IR across the resistor. Rewriting this expression, we have

$$\frac{dI}{V - IR} = \frac{dt}{L}$$

Integrating this, we get

$$log(V - IR) = -\frac{R}{L}t + \text{Constant}$$

Remembering that I=0 at t=0, it reduces to

$$I = \frac{V}{R}(1 - e^{-\frac{R}{L}t}) = I_o(1 - e^{-\frac{R}{L}t}) \tag{VI-B-2}$$

Equation (VI-B-2) represents the growth of the current in the LR circuit. Similarly, the emf equation for the circuit when the source is switched off and the current is decaying is written as

$$L\frac{dI}{dt} + IR = 0 \tag{VI-B-3}$$

Rewriting and integrating this expression, we get

$$log\ I = -\frac{Rt}{L} + \text{Constant}$$

Substituting the initial conditions, $I = I_o = V/R$ at $t = 0$, we have

$$I = I_o e^{-\frac{Rt}{L}} = \frac{V}{R} e^{-\frac{Rt}{L}} \tag{VI-B-4}$$

#### 6.8.2.2 AN LR CIRCUIT WITH A A-C SUPPLY

Let the source voltage V and current I be of the form

$$V = V_o\ cos\ \omega t + \phi \tag{VI-B-5}$$

and

$$I = I_o\ cos\ \omega t \tag{VI-B-6}$$

Then

$$\frac{dI}{dt} = -\omega I_o\ sin\ \omega t$$

and the emf equation (VI-B-1) reduces to

$$I_o(R\ cos\ \omega t - \omega L\ sin\ \omega t) = V_o\ cos\ (\omega t +\ \phi)$$

Comparing the coefficients of $cos\ \omega t$ and $sin\ \omega t$, we have

$$tan\ \phi = \frac{\omega L}{R} \tag{VI-B-7}$$

$$V_o = I_o \sqrt{R^2 + \omega^2 L^2}] \tag{VI-B-8}$$

Equations (VI-B-5) and (VI-B-6) tell you that the applied emf always leads the current by $\phi$, where $\phi$ is given by Equation (VI-B-7).

Now the voltage across the inductor is

$$V_L = L\frac{dI}{dt} = \omega L I_o\ cos\ (\omega t + \pi/2) \tag{VI-B-9}$$

which shows that $V_L$ *leads* the current I by $\pi/2$.

The magnitude of reactance of the inductor $(X_L)$ and the total impedance in an LR circuit (Z) can be easily calculated using the peak (or rms) values of current and voltage. From Equation(VI-B-9)

$$X_L = \frac{V_L}{I_o} = \frac{\omega L I_o}{I_o} = \omega L \tag{VI-B-10}$$

Similarly,

$$|Z| = \frac{V_o}{I_o} = \frac{I_o\sqrt{R^2 + \omega^2 L^2}}{I_o} = \sqrt{R^2 + \omega^2 L^2} \tag{VI-B-11}$$

You can arrive at the same results if you used the alternative method of representation in a-c, namely, by complex quantities (Appendix IV-B). The analysis can be found in most standard text-books and is similar to the one carried out for an RC circuit in Appendix IV-B. What you would get is

$$Z = \frac{V}{I} = R + i\omega L = \sqrt{R^2 + \omega^2 L^2} e^{i\theta} = |Z| e^{i\theta}$$

where $tan\ \theta = \omega L/R$

VII

# CHAPTER 7

# 7. Phase Measurement by Superposition

## 7.1 Introduction

The method of vector diagrams for determining the magnitudes and relative phases of voltages and currents in a-c networks is not easy in many cases. Consider, for example, the following circuit.

If you want to know the phase difference between the voltages across $R_1$ and $R_2$, it would not at all be a simple matter to infer this from a vector diagram. (Try it). For this reason, we introduce a method by which you can determine phase differences *directly* and study phase relationship in various networks.

## 7.2 Principle of Measurement

The principle we shall employ is *superposition* of the voltage to be measured and a *fixed standard voltage* so that phase is determined relative to that of the standard voltage. We shall refer to this superposition as 'mixing'.

An essential condition for this is that the fixed standard voltage must be *coherent* with the voltage under study. This means that the phase difference between the two must not change with time. In this board the output of a transformer serves as the coherent source, because this and the circuit under study are both governed by the a-c mains, which make the parent source. If you work, for instance, with two separate audio oscillators, the condition of coherence will not be satisfied even if the frequencies are matched very closely.

Suppose we wish to measure the phase of a signal $V_1$ relative to the standard signal $V_S$ [Fig.7.2(a)]. Assuming $|V_1| > |V_S|$, the signal $V_1$ is connected to a potential divider * and the output point P is adjusted to give exactly the same magnitude V as the standard potential $V_S$. We can now proceed to measure the phase of V relative to $V_S$, which will also be the phase of $V_1$ relative to $V_S$. To determine $\phi$ we mix the two voltages so that we get either $V$+$V_S$ or $V$-$V_S$. This

*The resistance of the potential divider should be sufficiently high (it is about 100$k\Omega$ in the network designed by us) so that it does not disturb the phase or magnitude of the voltage measured.

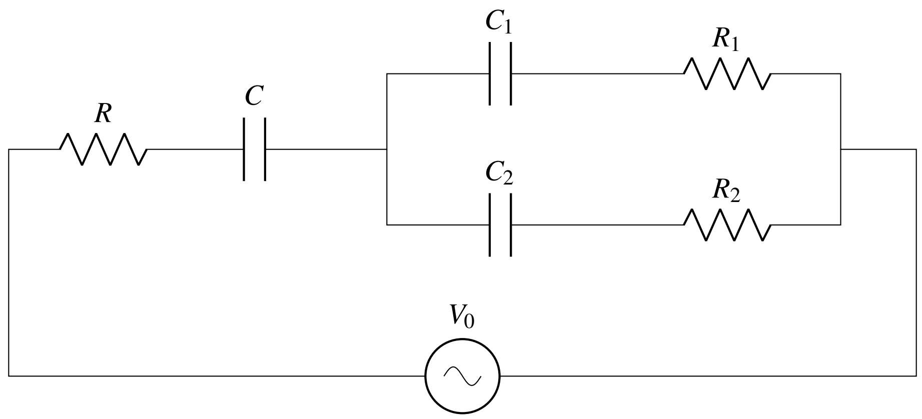


Figure 7.1

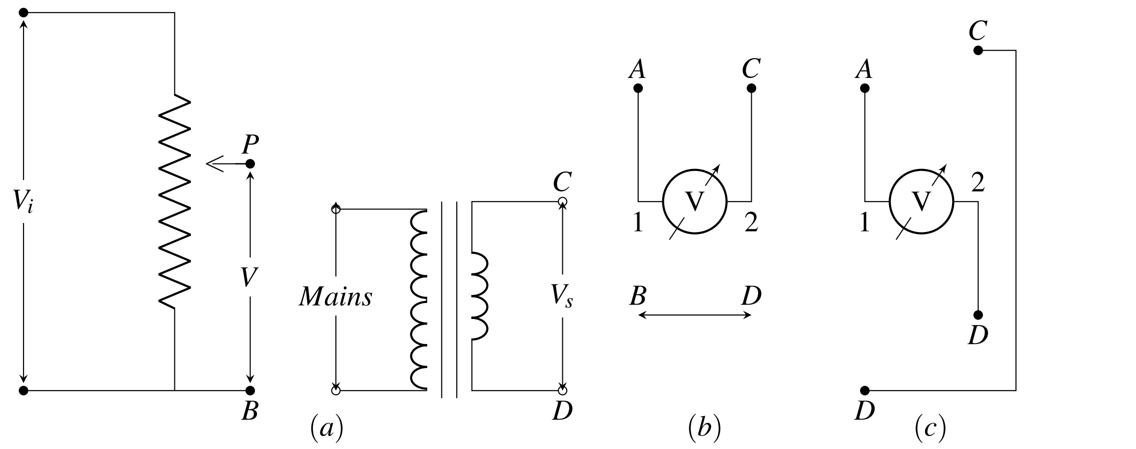


FIGURE 7.2 (a) An arrangement to achieve $|V| = |V_S|$ and mix the two voltages (b) and (c) voltage measurement for superposition and reversal of polarity of one of the voltages.

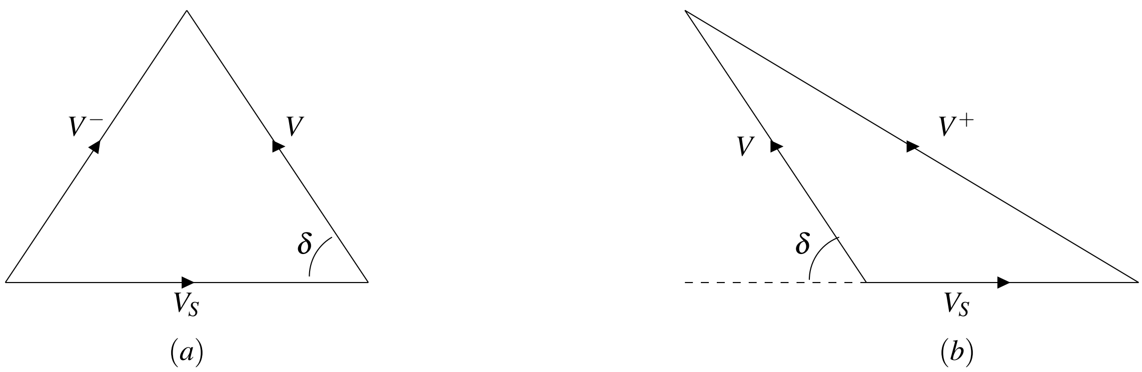


FIGURE 7.3

can be easily done by first mixing the voltages in one way as in Fig. 7.2(b) and then reversing the polarity of one of the two voltages for mixing [fig.7.2(c)]. On vector diagrams the two cases are illustrated in Fig. 7.3. The magnitudes of resultant voltage in the two cases are called $V^+$ and $V^-$.

We now measure the magnitudes of the resultant in the two cases. From Fig. 7.3 it can be seen (derive these results yourself) that

$$V^- = 2V \sin \frac{\delta}{2}$$

and

$$V^+ = 2V \cos \frac{\delta}{2}$$

so that

$$\frac{V^-}{V^+} = \tan \frac{\delta}{2} \tag{7.1}$$

This immediately gives us the phase difference $\delta$. If one signal has phase $\delta_1$ and another has phase $\delta_2$, their phase difference $\delta_1 - \delta_2$ gives the relative phase of the two signals, because both $\delta_1$ and $\delta_2$ are measured relative to a coherent standard.

There may be cases when the voltage to be measured is smaller than the standard voltage. In this case, we *drop the standard voltage* across the potential divider until it has the same magnitude as the voltage to be measured and then the mixing is carried out.

It may be noted that since the relative polarity of the transformer giving us $V_S$ and that which gives the source voltage $V_o$ to the network under study could be uncertain by $\pi$ in phase (terminals interchanged ), we are uncertain whether connection (b) or (c) in Fig. 7.2 gives $V^+$. Therefore $\delta/2$ obtained by Equation 7.2 is uncertain by $\pi/2$, whence $\delta$ is uncertain by $\pi$.

Yet further, Equation (7.1) involves only the magnitudes $V^+$, $V^-$ and leaves us uncertain about the sign of $\delta/2$, hence $\delta$.

The net result is that in interpreting the result we have to use other information to decide on the sign of $\delta$ and a possible shift of $\pi$ (which also means sign change for tan $\delta$)

## 7.3 The Network Board - 7

A schematic diagram of the lay out of the board is given below:

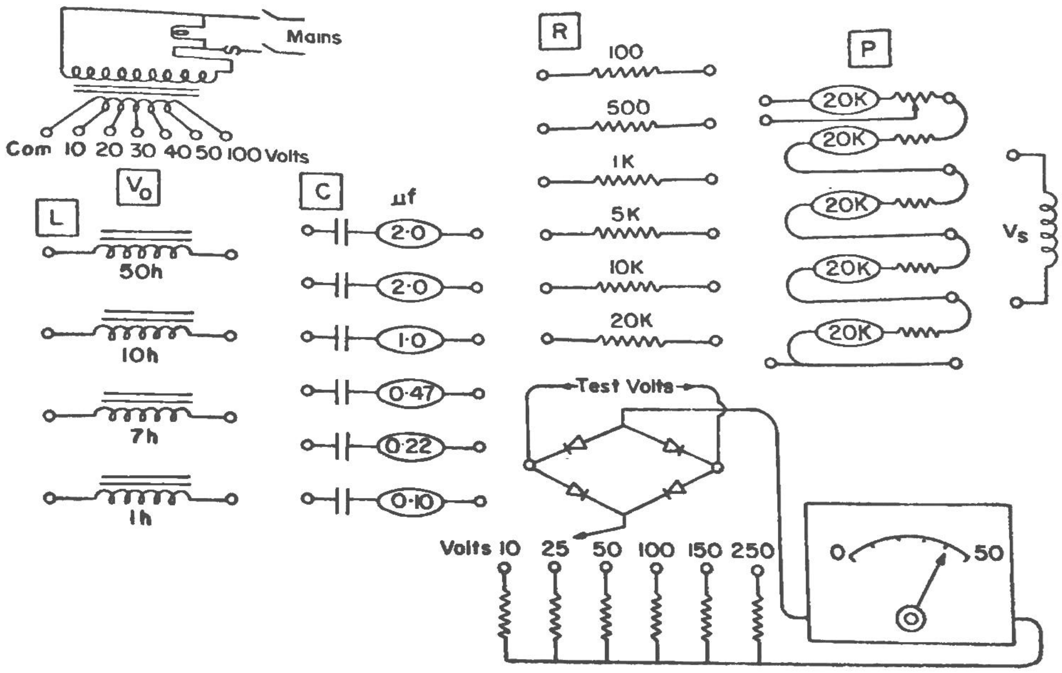


FIGURE 7.4

On the right is the coherent standard voltage $V_S$ from a transformer output (about 20 volts). There is a switch for reversing its polarity, i.e. for changing its phase by $\pi$.

The potential divider P consists of a set of four resistors of equal value ($20k\Omega$) in series connected to a variable pot ($20k\Omega$) so that a continuous adjustment of voltage is possible.

Then there is provided a whole range of L, C and R values for making any network for study. Another transformer provides different voltages for applying to this. The voltmeter is shown at the bottom right with its bridge circuit and range selector.

## 7.4 Experiments

### 7.4.1 Phase of voltages across resistors and capacitors

#### Experiment VII-A: To study the relative phases of voltages across resistors and capacitors in series.

A supply of about 100 volts is connected to a series of resistors. Choose a voltage across one of the resistors (conveniently about 30 volts) and measure its phase in the manner suggested; i.e. by dropping to the 'standard voltage' and mixing it. Repeat this for the other resistors. What

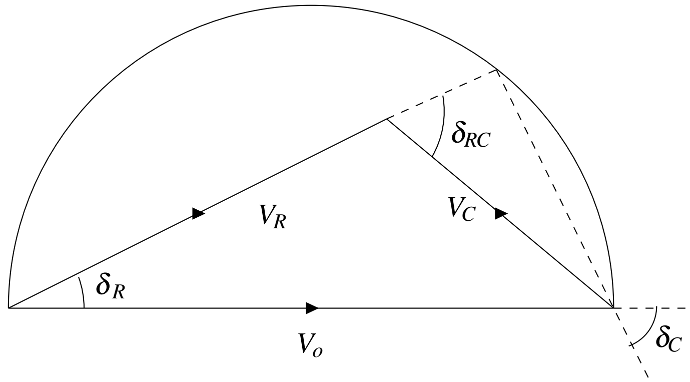


FIGURE 7.5

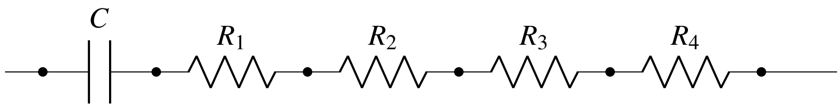


FIGURE 7.6

information do you get about the phase relationships amongst the voltages across the various resistors?

Now do the same for a number of capacitors in series. Interpret your results.

### 7.4.2 Phases of $V_R$ and $V_C$ in RC circuit

#### Experiment VII-B: To measure the phase difference between $V_R$ and $V_C$ in a simple RC circuit.

Connect a resistor and a capacitor in series to a source $V_o$. Measure the phase difference between $V_R$ and $V_C$ and that between $V_C$ and $V_S$. If the polarities of the two transformers (feeding $V_S$ and $V_o$) match, then the phase of $V_S$ is same as that off $V_o$. Therefore the phases measured are those between $V_R$ and $V_o$ and between $V_C$ and $V_o$. Let us call them $|\delta_R|$ and $|\delta_C|$ respectively. Since the signs of $\delta_R$ and $\delta_C$ are unknown, you may find it difficult to decide whether the sum or difference of $|\delta_R$ and $|\delta_C|$ gives us $|\delta_{RC}|$, the phase difference between $V_R$ and $V_C$. But Fig. 7.5 helps to give us

$$|\delta_{RC}| = |\delta_R| + |\delta_C$$

If the polarities of the two transformers are interchanged, it will be equivalent to interchanging $V^+$ and $V^-$ in Equation (7.1), and thus getting us $|\delta_R|$ and $|\delta_C|$ values which are off from the earlier values by $\pi$ *each*, so that $|\delta_{RC}|$ will be off by $2\pi$. In phase measurements this means that $|\delta_{RC}|$ is not affected at all.

Thus the only care needed in the work is that in the measurement of both $\delta_R$ and $\delta_C$ the mix with the same setting [(b) or (c) in Fig. 7.2] should be designated as $V^+$.

### 7.4.3 More about phase relationships in RC circuit

#### Experiment VII-C: To study more about phase relationships in an RC network.

With a capacitor and a set of resistors in series measure the phases (relative to the standard) of $V_C$, $V_{CR_1}$, $V_{CR_{12}}$ etc.

Represent them by vectors starting from the same point. See also if you can construct a polygon with the vectors. You can take a number of capacitors in series with a resistor and repeat this experiment. Now you can make measurements on capacitors and resistors connected in different ways. For instance, measure the phase difference between $V_{in}$ and $V_{out}$ in the circuit given below. Interpret your results.

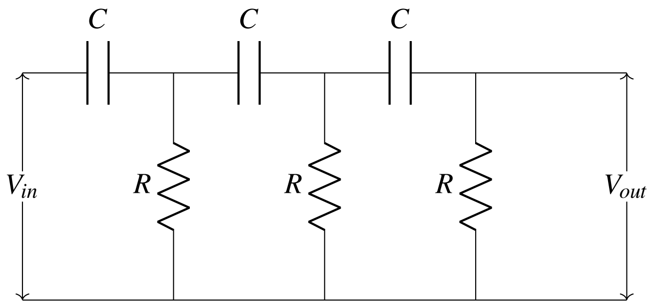


FIGURE 7.7

The RC combination is sometimes used for shifting the phase of a signal (see the phase shift oscillator in Appendix IV-A). In the RC network (Fig. 7.7), it can be shown that a phase shift of $180^o$ occurs for a frequency

$$f = \frac{1}{2\pi RC\sqrt{6}}$$

For 50 hz, the value of RC turns out to be about 1.3 msec. Choose $1\mu$f capacitors and a value of R around 1.3kΩ and make up such a circuit. Now measure directly the phase difference between $V_{in}$ and $V_{out}$. In addition to the phase shift there is also attenuation of the signal so that if $V_{in}$ is 220 volts, $V_{out}$ will turn out to be only about 7 volts.

You may also measure the phase differences between $V_{in}$ and the voltages after the first pair CR (i.e. across the first resistor) and after the second pair CR. See if the phase changes by $60^o$ each time. (It should not!)

If you measure the phase difference between $V_C$ and $V_R$, say the first pair following the input, you will find that it is not $90^o$ as you may imagine at first glance. Can you explain this?

## 7.4.4 Phase relationships in LR circuit

### Experiment VII-D: To study the phase relationships in an LR circuit.

Connect an inductor and a resistor in series to a source $V_o$. Measure the phase difference between $V_o$ and $V_L$, $V_o$ and $V_R$. You can determine, from this, the phase difference $\delta$ between $V_L$ and $V_R$.. This will not be $\pi/2$ since there is power loss in the inductor. You can easily show (try this) that $\delta$ is given by

$$tan\left(\frac{\pi}{2} - \delta\right) = \frac{r}{\omega L}$$

where r is the effective power loss resistance of L. Compare the value r you obtain this way to the value obtained by triangulation (as in Experiment 6-D). An agreement within a factor of two is to be considered satisfactory.

The phase difference between $V_o$ and $V_R$ [you can derive it easily from the vector diagram in Fig. 6.3(b)] is given by

$$tan^{-1}\frac{\omega L}{R+r}$$

Verify this for a large number of combinations.

## 7.4.5 Phase of $V_R$ in LCR circuit

### Experiment VII-E: To study the phase of $V_R$ in an LCR circuit.

Connect an LCR circuit in series. Measure the phase difference between $V_R$ and $V_o$. This phase difference is given by

$$tan^{-1}\frac{\omega^2 LC - 1}{\omega C(R+r)}$$

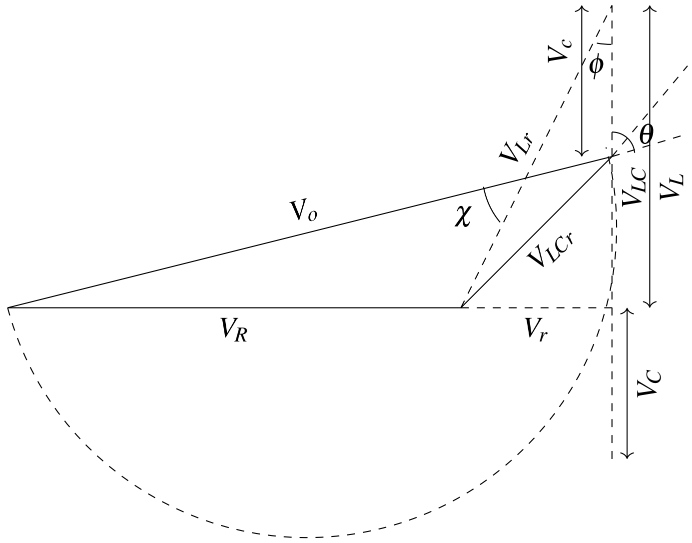


FIGURE 7.8

and will be zero at resonance. (You may recall that in the vector diagram, Fig. 6.17(a), $V_R$ becomes parallel to $V_o$). Verify this by varying C and study how $\delta_{OR}$ changes as you pass through resonance, i.e. as you go from

$$LC < \frac{1}{\omega^2} \text{ to } LC > \frac{1}{\omega^2}$$

### 7.4.6 Phases of various voltages in LCR circuit

#### Experiment VII-F: To study the phase relationships amongst various voltages in an LCR circuit.

In an LCR circuit, measure the phase differences $\delta(V_o, V_{Lr})$ and $\delta(V_o, V_C)$. Hence determine $\delta(V_C, V_{Lr})$. Do this for various values of C. Check if $\delta(V_C, V_{Lr})$ is independent of C. You may also infer the phase difference $\delta(V_o, V_L)$, though it is not directly measurable (because of r).

Figure 7.8 gives the full line triangle for voltages $V_o$, $V_R$ and $V_{LC}$, while the phase angles measured are $\delta(V_o, V_{Lr}) = \chi$ and $\delta(V_o, V_C) = \theta$. The deduced angle is $\delta(V_o, V_{Lr}) = \phi = \theta - \chi$ [Compare this with Fig. 6.2(b)].

From Fig. 7.8 we note that

$$tan\ \delta(V_o, V_C) = tan\ \theta = \frac{I_o(R+r}{I_o(\omega L - \frac{1}{\omega C})} = \frac{(R+r)}{(\omega L - \frac{1}{\omega C})}$$

and

$$tan\ \phi = \frac{I_o r}{I_o\ \omega L} = \frac{r}{\omega L}$$

From these expressions, one can deduce the expression for $\delta(V_o, V_{Lr}) = \chi$ since $\chi = \theta - \phi$.

### 7.4.7 Phase of any voltage in complex networks

#### Experiment VII-G: To measure phase of the voltage across any two points in a complex network.

Measure the phase difference between $V_A$ and $V_B$ in the network shown in Fig. 7.1. You may try other networks where it is very difficult to measure the phase of the voltage across any two points in the network by the method of vector diagrams. In all such cases you will be able to measure it by this method directly.

VIII

# CHAPTER 8

# 8. Study of Electromagnetic Induction

## 8.1 Introduction

The basic principle of generation of alternating emf is electromagnetic induction was discovered by Michael Faraday.* This phenomenon is the production of an induced emf in a circuit (conductor) caused by a change of the magnetic flux linking the circuit. Faraday's law of induction tells us that the induced emf e is given by

$$e = -\frac{d\phi}{dt} \tag{8.1}$$

where $\frac{d\phi}{dt}$ represents the rate of change of flux linking the circuit. If you use mks units, e will be in volts, the flux in webers and t in sec. If, on the other hand you use Gaussian units, $\phi$ in gauss $cm^2$, then Equation (8.1) will read

$$e = -\frac{1}{c}\frac{d\phi}{dt}$$

where c is the speed of light in cm/sec. and e will then be in abvolt $(= 10^{-8} volt)$.

For a discussion of the concept of flux and Faraday's law turn to Appendix VIII-A. The experiments described in this chapter will further help you to understand the phenomenon.

## 8.2 The apparatus

It consists of a permanent magnet mounted on an arc of a semi-circle of radius 50 cm. The arc is part of a rigid frame of aluminium and is suspended at the centre of the arc so that the whole frame can oscillate freely in its plane (Fig. 8.1). Weights have been provided on the diagonal arm, so that by altering their positions the time period of oscillation can be varied from about 1.5 to 3 sec. Two coils of about 10,000 turns of copper wire loop the arc so that the magnet can pass freely through the coil. The two coils are independent and can be connected either in series or in parallel.

*It is said that when Faraday was asked about the use of his discovery he replied "what is the use of a new born baby?" Had Faraday made the discovery in modern days, he would probably have been asked "what is the relevance of your discovery?" indicating the great progress we have made in the nuances of language; we are, however, not quite sure what Faraday would have replied.

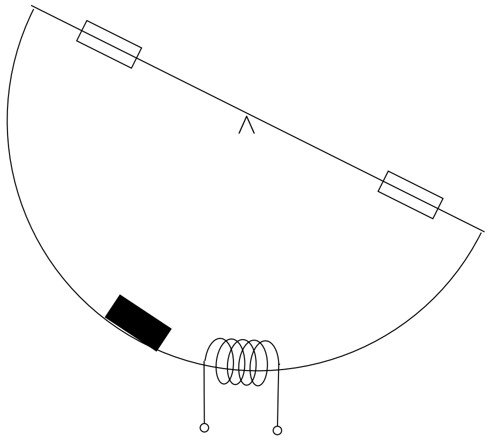

FIGURE 8.1

The amplitude of the swing can be read from the graduations on the arc. Plate III shows a photograph of the complete unit. When the magnet moves through and out of the coil, the flux of the magnetic field through the coil changes, inducing the emf.

In order to measure this emf, we resort to the now familiar trick of charging a capacitor through a diode and measuring the voltage developed across the capacitor at leisure (Fig. 8.2).

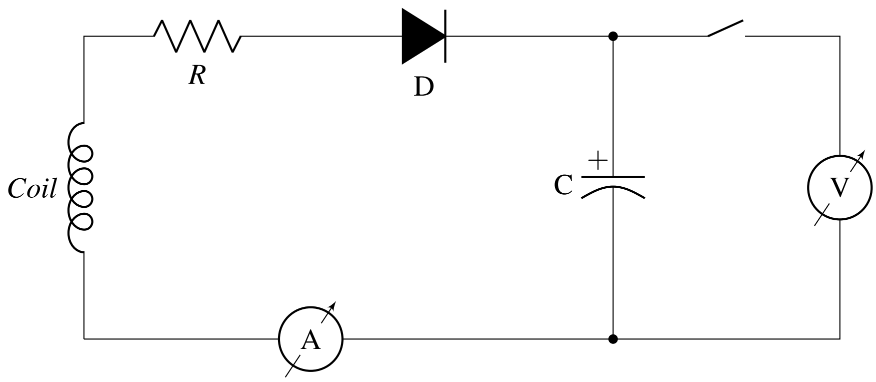


FIGURE 8.2

R represents the coil resistance (about 500 ohm) plus the forward resistance of the diode. (If you introduce an additional resistance, that will also have to be included in R). The capacitors used are in the range of $100\mu$f and the charging time RC is of the order of 40 msec. It will turn out that this time is somewhat larger than the time during which the emf in the coil is generated so that the capacitor does not charge up to the peak value in a single swing and may take about 10 oscillations to do so. This may be checked by the current meter in the circuit which will tell you when the charging current ceases to flow.

The peak value of the emf generated may also be measured in a way that has been discussed in Chapter IV. This is the null method in which one compares the varying emf with a d-c voltage. The arrangement is shown in Fig. 8.3. The voltmeter will record a 'kick' if the voltage across AB (potential divider) is smaller than the peak voltage developed across the coil so that all that is required is to increase the d-c voltage until the meter ceases to show any deflection.

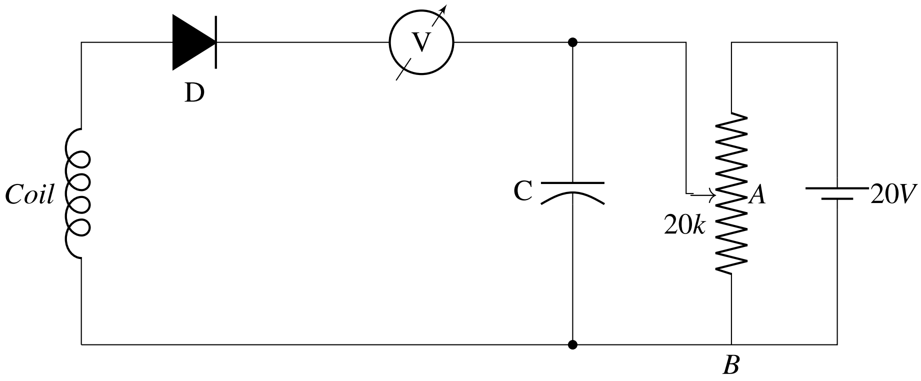


FIGURE 8.3

The part played by the capacitor is purely nominal. See if there is any difference in the performance without it.

In the experiments, try to measure the induced emf by both the methods suggested above.

## 8.3 Experiments

### 8.3.1 Induced EMF as a function of velocity of magnet

#### Experiment 8-A: To study the EMF induced as a function of the velocity of the magnet.

The magnet is placed at the middle of the arc. As the magnet starts far away from the coil, moves through it and recedes, the magnetic field through the coil changes from a small value, increases to its maximum and becomes small again thus inducing an emf (Appendix VIII-A).

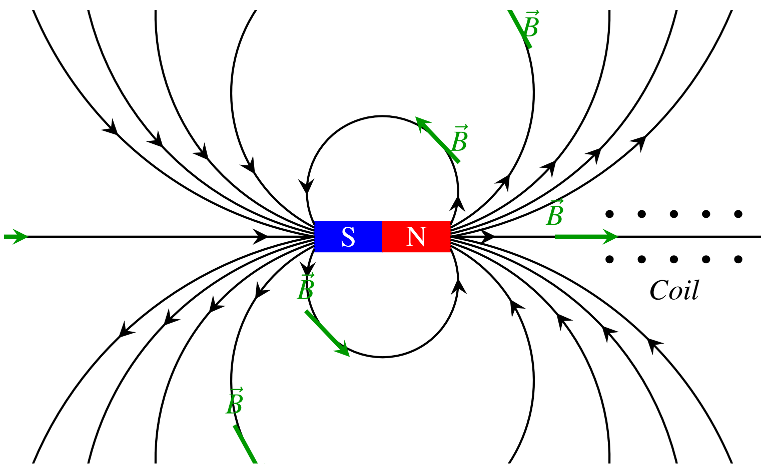


FIGURE 8.4. The magnetic field at the coil increases as the magnet approaches the coil

Actually, there is a substantial magnetic field at the coil only when it is very near the magnet; moreover, the speed of the magnet is largest when it ap approaches the coil since it is approximately in the mean position of the oscillation. Thus the magnetic field changes quite slowly with time when the magnet is far away and rapidly as it approaches the coil. Fig. 8.5 shows roughly the way $\vec{B}$ (at the coil) changes with time in this experiment. The centre of the hump refers to the time when the magnet is in the coil and the flat portion at the top corresponds to the finite length of the magnet.

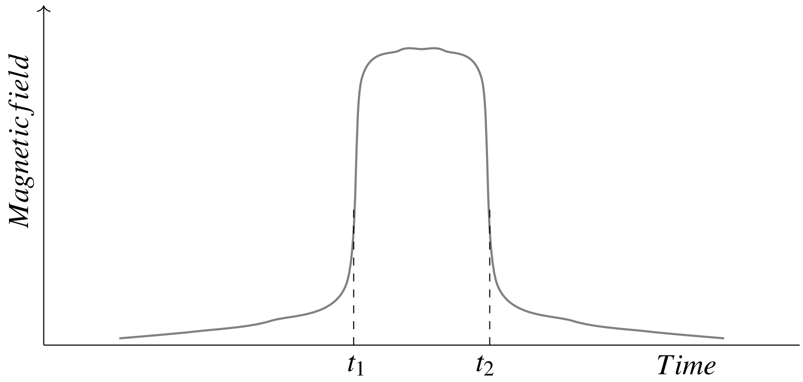


FIGURE 8.5 Variation of B at the coil

The time scale in this figure will depend on the amplitude of back-and- forth oscillation (as also the period) of the magnet. Figure 8.5 also represents the variation of $\phi$ with time because only a constant (*effective area*) relates with B. (The constant roughly equals the product of number of turns and area of the coil).

Now the induced emf is proportional to $d\phi/dt$. This slope in Fig. 8.5 is largest at times $t_1$ and $t_2$, so magnitude of e is largest at these times. But Lenz's law gives minus sign (—) in Equation (8.1), which means that e is negative when $\phi$ is increasing (at $t_1$) and positive when $\phi$ is decreasing (at $t_2$).

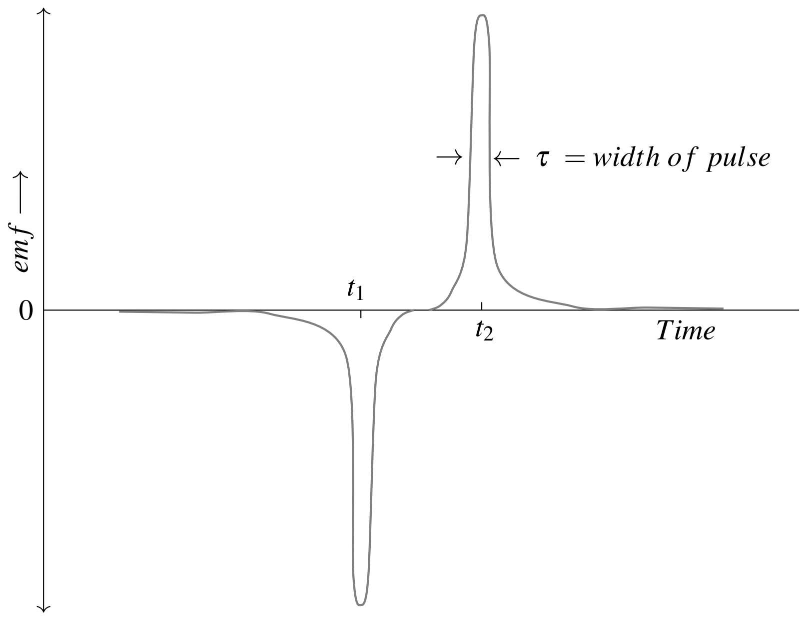


FIGURE 8.6 Variation of the induced emf with time.

This enables us to deduce Fig. 8.6 relating e with t.

Remember, this sequence of two pulses, one negative and one positive, occurs during just half a cycle. On the return swing of the magnet, they will be repeated. (Which one will be repeated first, the negative or the positive pulse?)

Consider now the effect of these pulses on the charging circuit of Fig. 8.2. The diode will conduct only during the positive pulse; at the first half swing, the capacitor charges up to a potential $e_1$ given by $\frac{1}{RC}\int e\,dt$, the integration being over the first pulse. During the next half swing, the diode will be cut off until the positive pulse reaches $e_1$ and then capacitor will be allowed to charge up to a slightly higher potential (say) $e_2$, and so on. In a few oscillations the capacitor will be charged up to the peak value $e_o$ of the voltage pulse. This will be indicated by the fact that the milliammeter will stop showing any kicks..

To get an estimate of $e_o$ we note that the equation for e can be put in the form

$$|e| = \frac{d\phi}{dt} = \frac{d\phi}{d\theta}.\frac{d\theta}{dt} \tag{8.2}$$

The first term $d\phi/d\theta$ depends on the magnet and the coil geometry alone. (Note that at $\theta$=0, the middle position, we have $\phi$ maximum. But our interest is in $d\phi/d\theta$, which is actually zero at $\theta$=0. ) The second term $d\theta/dt$ is deduced from the oscillation equation:

$$\theta = \theta_o \; sin \; 2\pi\frac{t}{T}$$

$$\frac{d\theta}{dt} = \frac{2\pi\theta_o}{T} \; cos \; 2\pi\frac{t}{T} \tag{8.3}$$

The peak voltage $e_o$ in the induced emf pulse corresponds to $(d\phi/dt)_{max}$. We now note from Fig. 8.5 that $(d\phi/dt)_{max}$ occurs at positions not far from the middle position. In Equation 8.3 the cosine does not differ much from unity for angles close to $2n\pi$. Hence we conclude that

$$|e_o| = \left(\frac{d\phi}{dt}\right)_{max} \approx \left(\frac{d\phi}{d\theta}\right)_{max} \frac{2\pi\theta_o}{T} \tag{8.4}$$

The important thing Equation (8.4) tells is that for the given magnet and coil system $e_0$ is proportional to the swing amplitude $\theta_0$ and to the reciprocal of the period T.

By repeating the experiments with different $\theta_0$ try if $e_0$ is proportional to $\theta_0$. By sliding the weights (Fig. 8.1), you may change T. Check if $e_0$ proportional to $1/T$.

Fig. 8.7 shows a plot of observed $e_0$ against the maximum velocity

$$\left[\left(\frac{2\pi\theta_0}{T}\right) XradiusR_0\right]$$

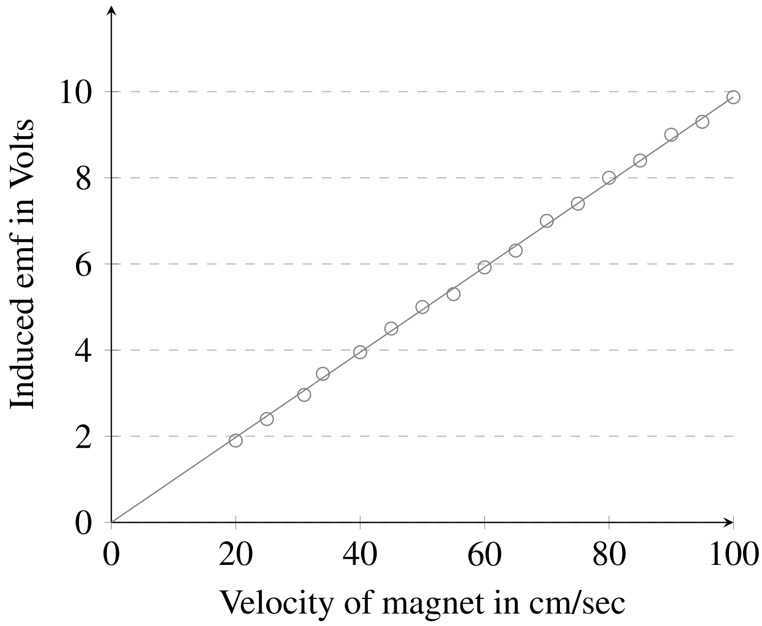


FIGURE 8.7

By repeating the experiments with different $\theta_0$ try if $e_0$ is proportional to $\theta_0$. By sliding the weights (Fig. 8.1), you may change T. Check if $e_0$ proportional to $1/T$.

### 8.3.2 Charge delivered due to induction

#### Experiment VIII-B: To study the charge delivered due to induction.

When the charging time RC of the capacitor is large compared with the pulse-width, the charge collected in one pulse (positive) is given by

$$q_1 = \frac{1}{R}\int_0^t e(t)dt = -\frac{1}{R}\int_0^t \frac{d\phi}{dt}dt = -\frac{1}{R}\int d\phi \tag{8.5}$$

Looking at Figs. 8.5 and 8.6 we see that the positive pulse corresponds to $\phi$ changing from the maximum (say $\phi_{max}$ ) to zero. Hence Equation (8.5) leads to

$$q_1 = \frac{\phi_{max}}{R} \tag{8.6}$$

The corresponding gain of voltage by the capacitor is

$$V_1 = \frac{\phi_{max}}{RC} \tag{8.7}$$

Now, arrange to keep RC large and find the voltage acquired by the capacitor in different numbers of positive pulses. See if the acquired voltage V comes out proportional to the number of positive pulses. [It will only hold until $V \ll e_0$. See Appendix III-B]

Check also if V acquired in a given number of positive pulses is proportional to $1/R$.

Next, from observations deduce $V_1$ which gives $\phi_{max}$ if RC is known. If the coil has m turns and its area is A, we have,

$$\phi_{max} = B_{max}.A.m$$

with V in volt, R in ohm, C in farad, Equation (8.8) gives $\phi_{max}$ in Weber. If A is taken in $metre^2$, you get $B_{max}$ in $Weber/metre^2$. As mentioned in Experiment 8-A, the diode allows the capacitor to charge only for the positive pulse (Fig. 8.6). You may arrange two sets of charging circuits as in Fig. 8.8 so that one capacitor charges up on the positive pulse and the other on the negative pulse. Verify that the charges on the capacitors are nearly the same. (What will the voltages on the capacitor depend on ?)

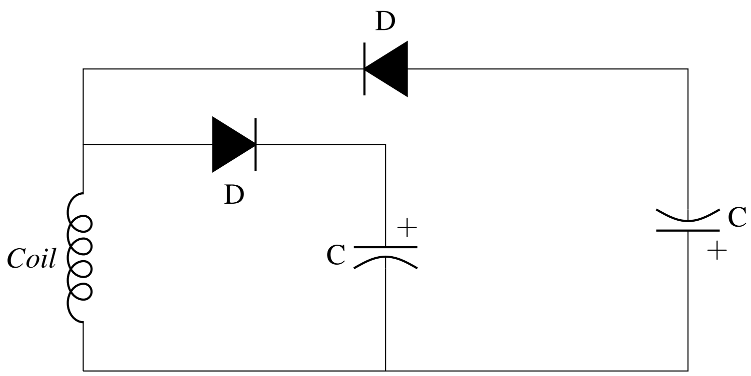


FIGURE 8.8

If you stop the oscillation (by hand) after a quarter oscillation (from the extreme position of the magnet to its mean position), only one capacitor will charge up. Try and find out if the sign of the emf induced is as should be according to Faraday's law.

### 8.3.3 Electromagnetic damping

#### Experiment VIII-C: To study electromagnetic damping.

We have, so far, neglected damping of the oscillations of the magnet. Successive oscillation will not be of the same amplitude as you can check from a rough measurement of the amplitude after, say, 10 oscillations.

There are many reasons for this damping. There is always some air resistance; then again the system is not really free from friction at the point of suspension. But the most important (and interesting !) source is that the induced emf in the coil itself introduces a damping through a mechanism which goes by the name of Lenz's law. This law states that the direction of the induced emf is always such as to oppose the change that causes it. Here the motion of the magnet causes the emf. Therefore any current produced by this emf must interact with the magnet so as to oppose that motion. Note that emf does *not* interact with the magnet, *current* does that. So the opposing action will depend on the induced current. The energy dissipation will not be the same after each oscillation ; in fact as a rule in oscillatory systems, the fractional loss of energy turns out to be roughly constant.

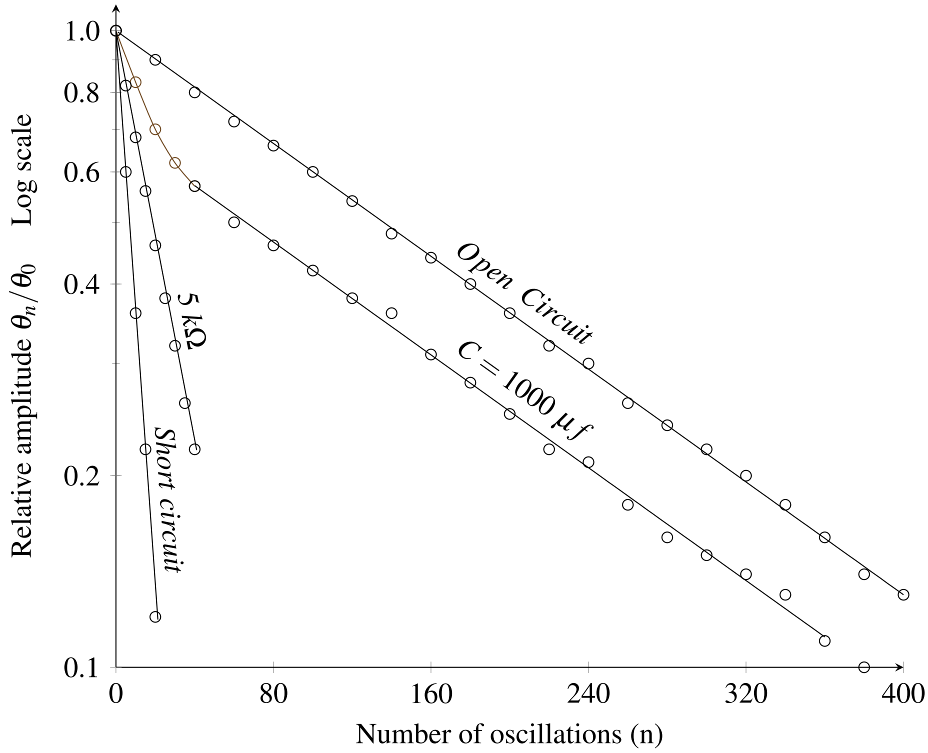


FIGURE 8.9

That is to say, suppose the energy of the system is $E_n$ after n oscillations. Then,

$$\frac{E_n}{E_{n-1}} = a \tag{8.8}$$

where a is nearly independent of n. It is easy to see that this implies

$$\frac{E_n}{E_0} = a^n \tag{8.9}$$

where $E_o$ is the energy at the beginning.

Thus if you plot a graph of $log\ \theta_n$ as a function of n you ought to get a straight line. In practice, you will find that this is only approximately true. Since the energy is proportional to the square of the amplitude, Equation (8.8) gives

$$\frac{\theta_n}{\theta_o} = \sqrt{\frac{E_n}{E_o}} = a^{n/2} \tag{8.10}$$

First, keep the coil open circuited. Fix the initial amplitude of the magnet and measure the amplitude S after a number of oscillations. You will find that the amplitude is still considerable even after 200 oscillations, since in this case there is no electromagnetic damping at all. Plot $log\ \theta_n$ as a function of n.

Next, try the same with a short circuited coil. This time the amplitude diminishes rapidly and about 20 oscillations or so are all that give measurable amplitudes. Again plot $log\ \theta_n$ vs n. You may also connect a finite load such as a 500 ohm resistor and make the same measurements. Finally, try also a big capacitor, say 2000 $\mu$f, as a load. At each swing the capacitor keeps charging up and energy has to be supplied to build up this energy, as well as the energy that will be lost through leakage. Try and interpret the graphs that you obtain in these cases; the case of a capacitative load is somewhat complicated. Fig. 8.9 shows these curves plotted from experimental data.

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## 8.4 APPENDIX

### 8.4.1 Appendix VIII-A FLUX OF THE FIELD AND FARADAY'S LAW

As pointed out at the beginning of this chapter, the concept of flux of the field is vital to the understanding of Faraday's Law.

Consider a small element of area $\overrightarrow{d\sigma}$. We assign a direction to this element by taking it to be normal to the plane of the area directed such that if it is bounded by a curve as shown in Fig. VIII-A (1), then the normal comes out of the plane of the paper towards you, the reader. In other words, it is the same direction as the movement of the axis of a right handed screw rotated in the sense of the arrow on the curve.

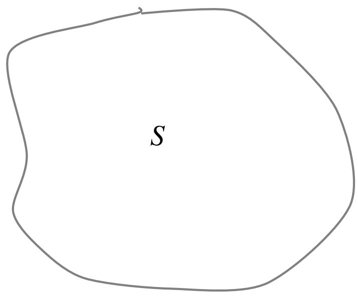


FIGURE VIII-A(1)

Suppose, now, this element of area is situated in a magnetic field $\overrightarrow{B}$.Then the scalar quantity

$$d\phi = \overrightarrow{B}.\overrightarrow{d\sigma} = |B||d\sigma|cos\ \theta \qquad \text{(VIII-A-1)}$$

is called the flux of $\overrightarrow{B}$ through area $\overrightarrow{d\sigma}$, where $\theta$ is the angle between the direction of magnetic field and the direction assigned to the area $\overrightarrow{d\sigma}$.

We can generalize this to define the flux over a finite area $\overrightarrow{S}$. In doing this, we must remember that the magnetic field $\overrightarrow{B}$ will not, in general, be the same at different points with in the finite area. We therefore divide up the area into small pieces, calculate the flux over each piece and integrate. Thus, the flux is

$$\phi = \int_s \overrightarrow{B}.\overrightarrow{d\sigma} \qquad \text{(VIII-A-2)}$$

where the symbol s signifies that we are to integrate over the entire $\overrightarrow{S}$ [Fig. VIII-A(2)]. Obviously , you cannot take $\overrightarrow{B}$ out of the integral in Equation (VIII-A-2) unless $\overrightarrow{B}$ is the same everywhere in $\overrightarrow{S}$.
If the magnetic field at every point changes with time as well, then the flux will also change with time.

$$\phi = \phi(t) = \int_S \overrightarrow{B(t)}.\overrightarrow{d\sigma} \qquad \text{(VIII-A-3)}$$

Faraday's discovery was that the rate of change of flux $d\phi/dt$ is related to the work done on taking a unit positive charge around the contour C [Fig. VIII-A(2)] in the reverse direction. This work done is just the emf.

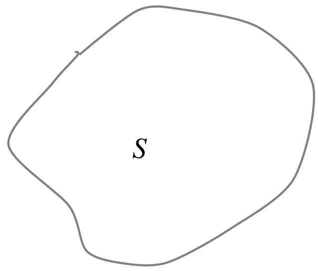


FIGURE VIII-A(2)

Accordingly, we can state Faraday's law in its usual form that the induced emf is given by

$$e = -\frac{d\phi}{dt} \tag{VIII-A-4}$$

If you look at Equation (VIII-A-3), you will see that even if $\overrightarrow{B}$ does not change with time the flux may still vary if the surface S is somehow changing with time. Consider, for example, a frame of wires ABCD, as drawn in Fig. VIII-A (3), situated in a constant magnetic field.

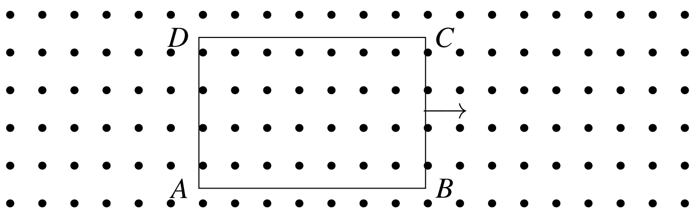


FIGURE VIII-A(3)

If the side BC is moved out thus increasing the area of the loop ABCD, the flux of $\overrightarrow{B}$ through the loop increases with time. Here also Faraday's law will apply as stated in Equation (VIII-A-4)†.

†There are, however, a number of situations in which Faraday's law would not hold. For a beautiful discussion, read Feynman's Lectures in Physics, Ch. 17, Vol II.

## LIST OF BOOKS FOR READING

The following is a partial list of books that you will find useful for understanding, in depth, the subjects we have dealt with.

R. P. FEYNMAN, The Feynman Lectures on Physics, Vol. I and II, Addison- Wesley, Reading and B. I. Publications, Bombay, 1969.

C. KITTEL, Mechanics, Berkeley Physics Course, Vol. I, Tata McGraw-Hill, Bombay.

E. M. PURCELL, Electricity and Magnetism, Berkeley Physics Course, Vol. II, Tata McGraw-Hill, Bombay.

F. S. CRAWFORD, Waves, Berkeley Physics Course, Vol. Ill, McGraw-Hill, New York, 1968.

F. EVDOKIMOV, Fundamentals of Electricity, Mir Publishers, Moscow, 1971.

T. KORNEFF, Introduction to Electronics, Academic Press, New York, 1966.

J. MILLMAN and C.C. HALKIAS, Integrated Electronics, McGraw-Hill, Koshiba, Tokyo,1972